Cosmology

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Tentative outline

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1. Introduction: homogeneous Universe
   1.1 Metric, redshift
   1.2 Friedmann equation and covariant energy conservation
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   1.4 Sample solutions
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   1.5 Stages of real Universe
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6. Inflation
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      axion misalignment

**Lecture 9**

7. Alternatives to inflation
   7.1 Bouncing Universe
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8. Conclusions
\hbar = c = k_B = 1 \text{ calculator:}

ppc.inr.ac.ru/eng/uc.php
Causal structure of space-time in hot Big Bang theory (i.e., assuming that the Universe started right from the hot epoch)

Angular size of horizon at recombination $\approx 2^\circ$. 
There are perturbations which were superhorizon at the time of recombination, angular scale $\gtrsim 2^\circ$. **Causality**: they could not have been generated at a hot epoch!
- 1000 K
- last scattering of CMB photons
- radiation-matter equality
- nucleosynthesis
- Generation of dark matter
- Generation of matter-antimatter asymmetry

Today

\[ z \approx 0.6: \text{accelerated expansion begins} \]

2.7 K

13.8 billion years

3000 K

9200 K

\[ 10^{10} - 10^9 \text{ K} \]

380 thousand years

70 thousand years

1 - 500 s

Question mark
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\[ T = 2.726^\circ K, \quad \frac{\delta T}{T} \sim 10^{-4} - 10^{-5} \]
Einstein:

\[- \Delta \Psi + 3 \frac{a'}{a} \Psi' - 3 \frac{a'^2}{a^2} \Phi = 4 \pi G a^2 \delta \rho_{tot} , \]

\[- \Psi' + \frac{a'}{a} \Phi = -4 \pi G [(\rho + p) v]_{tot} , \]

\[\Psi'' - \frac{1}{3} (\Delta \Phi + \Delta \Psi) + \frac{a'}{a} (2 \Psi' - \Phi') - 2 \frac{a''}{a} \Phi + \frac{a'^2}{a^2} \Phi = -4 \pi G a^2 [\delta p]_{tot} , \]

\[\Delta (\Phi + \Psi) = -12 \pi G a^2 [(\rho + p) \pi]_{tot} . \]

Covariant conservation:

\[\delta \rho' + 3 \frac{a'}{a} (\delta \rho + \delta p) + (\rho + p) (\Delta v + 3 \Psi') = 0 , \]

\[[(\rho + p) v]' + \delta p + (\rho + p) \left( 4 \frac{a'}{a} v + \pi + \Phi \right) = 0 . \]
Ideal fluids

Einstein eqs.:

\[ \Delta \Phi - 3 \frac{a'}{a} \Phi' - 3 \frac{a''}{a^2} \Phi = 4\pi Ga^2 \cdot \delta \rho_{tot} \]

\[ \Phi' + \frac{a'}{a} \Phi = -4\pi Ga^2 \cdot [(\rho + p)v]_{tot} \]

\[ \Phi'' + 3 \frac{a'}{a} \Phi' + \left( 2 \frac{a''}{a} - \frac{a'^2}{a^2} \right) \Phi = 4\pi Ga^2 \cdot \delta p_{tot} \]

Covariant conservation

\[ \delta \rho' + 3 \frac{a'}{a} (\delta \rho + \delta p) + (\rho + p)(\Delta v - 3 \Phi') = 0 \]

\[ [(\rho + p)v]' + 4 \frac{a'}{a} (\rho + p)v + \delta p + (\rho + p)\Phi = 0 , \]
Single component fluid

Master equation \( (u_s^2 = w) \)

\[
\Phi'' + 3 \frac{a'}{a}(1 + u_s^2)\Phi' + \left[ 2 \frac{a''}{a} - \frac{a'^2}{a^2} (1 - 3u_s^2) \right] \Phi + u_s^2 k^2 \Phi = 0 .
\]

Perturbation in energy density:

\[
4\pi G a^2 \delta \rho = - \left( k^2 \Phi + 3 \frac{a'}{a} \Phi' + 3 \frac{a'^2}{a^2} \Phi \right)
\]
Growth of perturbations (linear regime)

Radiation domination

\( t_{eq} \)

\( t_{rec} \)

\( t_{\Lambda} \)

\( t \)

Matter domination

\( \delta_\gamma \)

\( \delta_{DM} \)

\( \delta_B \)

\( \Phi \)

\( \Lambda \) domination
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\[-\Delta \Psi + 3 \frac{a'}{a} \Psi' - 3 \frac{a'^2}{a^2} \Phi = 4 \pi G a^2 \delta \rho_{tot} , \quad (1)\]

\[-\Psi' + \frac{a'}{a} \Phi = -4 \pi G a^2 [(\rho + p)v]_{tot} , \quad (2)\]

\[\Psi'' - \frac{1}{3} (\Delta \Phi + \Delta \Psi) + \frac{a'}{a} (2 \Psi' - \Phi') - 2 \frac{a''}{a} \Phi + \frac{a'^2}{a^2} \Phi = -4 \pi G a^2 [\delta p]_{tot} , \quad (3)\]

\[\Delta (\Phi + \Psi) = -12 \pi G a^2 [(\rho + p)\pi]_{tot} . \quad (4)\]

Covariant conservation:

\[\delta \rho' + 3 \frac{a'}{a} (\delta \rho + \delta p) + (\rho + p) (\Delta v + 3 \Psi') = 0 , \quad (5)\]

\[([(\rho + p)v]')' + \delta p + (\rho + p) \left(4 \frac{a'}{a} v + \pi + \Phi \right) = 0 . \quad (6)\]
Planck + all: Scalar tilt vs tensor power
BICEP-2 claim $r = 0.2$

$r = 0.2$ is large: 10% contribution to $\delta T$ at low multipoles, $l \lesssim 30$.

BICEP2 and Planck with $dn_s/d \log k = -0.02$ (very large)
Inflation: $dn_s/d \log k \approx -0.001$

Planck + all
BAO in power spectrum

(a) 2dFGRS+SDSS main

(b) SDSS LRG

(c) all

\[ \log_{10} \frac{P(k)}{P(k)_{\text{smooth}}} \]

\[ k / h \text{ Mpc}^{-1} \]
BAO in correlation function
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CMB temperature angular spectrum

Angular scale

Multipole moment, $\ell$

$D_\ell [\mu K^2]$
Three contributions
Top. defects are not seeds for perturbations

\[ l(l+1)C_l / 2\pi [\mu K^2] \]
Effects of adiabatic and entropy perturbations

adiabatic perturbations

entropy perturbation
Effect of baryons

$\Omega_b h^2 = 0.06$

$\Omega_b h^2 = 0.005$
Effect of curvature (left) and $\Lambda$
Einstein:

\[ -\Delta \Psi + 3 \frac{a'}{a} \Psi' - 3 \frac{a'^2}{a^2} \Phi = 4\pi G a^2 \delta \rho_{tot} , \quad (1) \]

\[ -\Psi' + \frac{a'}{a} \Phi = -4\pi G a^2 [(\rho + p)v]_{tot} , \quad (2) \]

\[ \Psi'' - \frac{1}{3} (\Delta \Phi + \Delta \Psi) + \frac{a'}{a} (2\Psi' - \Phi') - 2 \frac{a''}{a} \Phi + \frac{a'^2}{a^2} \Phi = -4\pi G a^2 [\delta p]_{tot} , \quad (3) \]

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\[ [(\rho + p)v]' + \delta p + (\rho + p) \left( 4 \frac{a'}{a} v + \pi + \Phi \right) = 0 . \quad (6) \]
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WMAP polarization sky
Planck polarization spots
WMAP cross correlation spectrum TE
Effects of scalar (left) and tensor (right) perturbations.
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Slow roll

Equations

\[ 3H \dot{\phi} = -V' \]

\[ H^2 = \frac{8\pi}{3} \frac{V}{M_{Pl}^2} \]

Parameters

\[ \varepsilon = \frac{V'^2 M_{Pl}^2}{16\pi V^2} \]

\[ \eta = \frac{V'' M_{Pl}^2}{8\pi V} \]
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   7.4 Generating scalar perturbations: conformal models

8. Overall conclusions
1. Consider single field inflation in slow roll regime, at which the slow roll parameter $\varepsilon$ decreases in time. Take any inflaton potential consistent with the CMB and galaxy distribution data. Show that during the period at inflation, which is responsible for generating the adiabatic perturbations, the inflaton field rolls down at least by

$$\Delta \phi \gtrsim 10r \cdot M_{Pl},$$

where $r$ is the tensor-to-scalar ratio. [This means, in particular, that the discovery of tensor modes with $r \sim 0.2$, as originally claimed by BICEP-2, would imply that the variation of the inflaton over the relevant period of time at inflationary epoch was super-Planckian.]
2. Relatively short gravity waves, created at inflation, after horizon re-entry at radiation domination can be viewed as a collection of gravitons (just like electromagnetic waves emitted by antenna can be viewed as a collection of photons). Assuming that the Hubble parameter $H$ some 60 e-foldings before inflation end is known, calculate the average (over ensemble of universes) number of gravitons $\langle N(k, \Delta k) \rangle$ in the present visible Universe in the interval of momenta from $k/a_0$ to $(k + \Delta k)/a_0$, and relative variance of this number

$$\sqrt{\langle N^2(k, \Delta k) \rangle - \langle N(k, \Delta k) \rangle^2 \langle N(k, \Delta k) \rangle}.$$ 

Dropping the assumption about the value of the Hubble parameter, calculate these quantities for the inflaton potential $V = m^2 \phi^2 / 2$. Give numerical estimates in the latter case for $k/a_0 = 1 \text{ Mpc}^{-1}$, $\Delta k = k$. 
3. Consider Minkowski space and generalized Galileon theory with the Lagrangian

\[ L = e^{4\pi} F(Y) + e^{2\pi} K(Y) \Box \pi \]

where

\[ Y = e^{-2\pi} (\partial \pi)^2. \]

Let \( F \) and \( K \) be chosen in such a way that there is a solution

\[ e^\pi = \frac{\text{const}}{-t}, \quad t < 0. \]

Let this solution be healthy, i.e., there are neither ghosts nor gradient instabilities among preturbations about this solution. Find the power spectrum of perturbations \( \delta \pi \) about this solution in the regime \( k|t| \ll 1 \), where \( k \) is the spatial momentum of a mode.