The Standard Model
and (some of) its extensions

R. Barbieri
GGI, Florence, January 9-27, 2017
Program

I. The SM and its status, as of 2016
II. Problems of (questions for) the SM

III. Minimal Mirror Twin Higgs (2 lectures)
IV. Anomalies in B-decays
V. Axion searches by way of their coupling to the spin
I. The Standard Model and its status summarized
The SM Lagrangian  
(since 1973 in its full content)

\[ \mathcal{L}_{SM} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + i\bar{\psi} \slashed{D}\psi \quad (\sim 1975-2000) \]

\[ + |D_\mu h|^2 - V(h) \quad (\sim 1990-2012) \]

\[ + \psi_i \lambda_{ij} \psi_j h + h.c. \quad (\sim 2000-\text{now}) \]

In () the approximate dates of the experimental shining of the various lines (at different levels)

The synthetic nature of PP exhibited
QCD in full strength

(see Maltoni's lectures)

\[ \alpha_s(Q^2) \]

\[ \text{QCD } \alpha_s(M_Z) = 0.1181 \pm 0.0011 \]

even though in the strong coupling regime...
**Precision in ElectroWeak Physics**

(a story that goes on from about 1970 on and still keeps its relevance)

<table>
<thead>
<tr>
<th>$\Delta \mathcal{O}/\mathcal{O}$</th>
<th>$APV$</th>
<th>$(g - 2)_e$</th>
<th>$(g - 2)_\mu$</th>
<th>$W, Z$</th>
<th>$m_{top}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-3}$</td>
<td>$10^{-8}$</td>
<td>$10^{-6}$</td>
<td>$10^{-(3/5)}$</td>
<td>$10^{-2}$</td>
<td></td>
</tr>
<tr>
<td>$d$(cm)</td>
<td>$10^{-5}$</td>
<td>$10^{-11}$</td>
<td>$10^{-13}$</td>
<td>$10^{-16}$</td>
<td>$10^{-16}$</td>
</tr>
</tbody>
</table>

precision at work at many different scales
The ante-LEP knowledge
(about 1970 – 1990)

Experiments:

- polarized eN scattering at $q^2 = O(1)\text{GeV}^2$
- Atomic Parity violation

\[
R_{\nu} = \frac{\sigma(\nu_{\mu} N \to \nu_{\mu} X)}{\sigma(\nu_{\mu} N \to \mu X)} \quad R_{\bar{\nu}} = \frac{\sigma(\bar{\nu}_{\mu} N \to \bar{\nu}_{\mu} X)}{\sigma(\bar{\nu}_{\mu} N \to \mu X)}
\]

$\sigma(\nu_{\mu} e), \sigma(\bar{\nu}_{\mu} e)$ elastic

$e^+e^- \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-$ at low $q^2$

W–mass measurements

Defining:

\[
\mathcal{L}_{q^2<<M_Z^2}^{NC} = 4 \frac{G_F}{\sqrt{2}} \rho J_{\mu}^{NC} J_{\mu NC}^{CN} \quad J_{\mu NC}^{CN} = J_{\mu}^3 - \sin^2 \theta_W J_{\mu}^{em}
\]

$\Rightarrow \rho \approx 1, \quad \sin^2 \theta_W \approx 0.22$ within few %
The ante-LEP knowledge

\[ \Rightarrow \rho \approx 1, \quad \sin^2 \theta_W \approx 0.22 \quad \text{within few \%} \]

Theory:

- at tree level \( \rho = 1 \) from Higgs being a doublet

\[ V(H) = |H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2 \]

\[ SO(4) = SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R} \]

Veltman 1977 + ...

"custodial symmetry"

Sikivie et al 1980

- at 1 loop two types of contributions:

1. top-bottom-Goldstone bosons

2. Only 2 \( \log m_h \) dependent (see below)
The ante-LEP knowledge

⇒ \rho \approx 1, \quad \sin^2 \theta_W \approx 0.22 \quad \text{within few \%}

Theory:

- at 1 loop two types of contributions:

1. top-bottom-Goldstone bosons

the “gaugeless” limit of the SM

\[ L_{kin}(\pi_i) = Z_2^{(+)} \left| \partial_\mu \pi^+ - g \frac{v}{\sqrt{2}} W^\mu_+ \right|^2 + \frac{Z_2^{(0)}}{2} \left( \partial_\mu \pi^0 - \frac{gv}{2 \cos \theta} Z_\mu \right)^2 \]

\[ \Delta \rho = 3x \]

\[ x = \frac{G_F m_t^2}{8 \pi^2 \sqrt{2}} \]

\[ \delta V_\mu(Z \rightarrow b \bar{b}) = -\frac{g}{\cos \theta_W} x \bar{b}_L \gamma_\mu b_L \]

\[ \frac{\pi^0}{t} \quad \frac{\pi^0}{b} \quad \frac{\pi^+}{t} \]

\[ \frac{?}{\pi^0} \]
The ante-LEP knowledge

- at 1 loop two types of contributions:

2. Only two $\log m_h$ dependent (see below)

\[
\Delta \rho = -\frac{3\alpha}{8\pi \cos^2 \theta_W} \log \frac{m_H}{M_Z},
\]

\[
\frac{\sqrt{2} G_F M_W^2}{\pi \alpha} \left(1 - \frac{M_W^2}{M_Z^2}\right) \equiv 1 + \Delta r = 1 + \frac{11\alpha}{24\pi \sin^2 \theta_W} \log \frac{m_H}{M_Z}
\]

(B in the Landau gauge)

Passarino, Veltman 1979
Antonelli et al 1980
Sirlin 1980
LEP (and not only LEP) at work
(from 1990 on)

The observables at the Z-pole and the W-mass
Assuming quark-lepton and flavour universality,

3 effective observables only

In terms of the vector/axial couplings of the Z to the fermion $f$

$$g_A^f = T_{3L}^f (1 + \frac{\epsilon_1}{2})$$

$$\frac{g_V^f}{g_A^f} = 1 - 4 |Q_f| s^2 (1 + \frac{\epsilon_3 - c^2 \epsilon_1}{c^2 - s^2})$$

and the W-mass

$$\Delta r = \frac{1}{s^2} (-c^2 \epsilon_1 + (c^2 - s^2) \epsilon_2 + 2s^2 \epsilon_3)$$

$$s^2 c^2 = \frac{\pi \alpha(M_Z)}{\sqrt{2} G_F M_Z^2}$$
Why this peculiar definition of the $\epsilon_i$?

$$\Pi_{ij}^{\mu\nu}(q^2) = -i \left[ A_{ij}(0) + q^2 F_{ij}(q^2) \right] \eta^{\mu\nu} + (q^\mu q^\nu - \text{terms})$$

with $i, j = W, Z, \gamma$ or $i, j = 0, 3$ for $B, W^3$

Defining:

$$\hat{T} = \frac{1}{m_W^2} (A_{33}(0) - A_{WW}(0)); \quad \hat{S} = \frac{c}{s} F_{30}(0); \quad \hat{U} = F_{WW}(0) - F_{33}(0)$$

$$\epsilon_1 = \hat{T} + \text{smaller oblique + non oblique}$$

$$\epsilon_2 = \hat{U} + \text{smaller oblique + non oblique}$$

$$\epsilon_3 = \hat{S} + \text{smaller oblique + non oblique}$$

non-oblique = vertices, boxes

$$\Pi_{WW}, \Pi_{33}, \Pi_{30}, \Pi_{00} \Rightarrow 8 \left( \Pi(0), \Pi'(0) \right)$$

$$8 = 2 \left( \Pi_{\gamma\gamma}(0) = \Pi_{\gamma Z}(0) = 0 \right) + 3 \left( g, g', v \right) + 3 \left( \hat{S}, \hat{T}, \hat{U} \right)$$

$U$ less UV-sensitive than $S$ and $T \Rightarrow$ only 2 independent $\log m_h$ terms
Constraining the top mass

La Thuile, April 1994
La Thuile, April 1994

SM fit compared with in the CDF paper of Sept 1994

For the Higgs boson a similar story in July 2012

**EW precision**

<table>
<thead>
<tr>
<th></th>
<th>ATLAS</th>
<th>CMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_h/GeV = 97^{+23}_{-17}$</td>
<td>$126.0 \pm 0.4 \pm 0.4$</td>
<td>$125.3 \pm 0.4 \pm 0.5$</td>
</tr>
</tbody>
</table>

$\Delta_{Altarelli, B}$
Current SM predictions (all OK with exp)

\[ g, \ g', \ \nu \quad + \quad g_S, \ m_t, \ m_h, \ \Delta \alpha_{\text{had}} \]

\[ G_\mu = 1.1663787 \times 10^{-5} \ \text{GeV}^{-2} \]
\[ \alpha = 1/137.035999139 \]

<table>
<thead>
<tr>
<th>Prediction</th>
<th>( \alpha_s )</th>
<th>( \Delta \alpha_{\text{had}}^{(5)} )</th>
<th>( M_Z )</th>
<th>( m_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_W ) [GeV]</td>
<td>80.3618 ± 0.0080</td>
<td>±0.0008</td>
<td>±0.0060</td>
<td>±0.0026</td>
</tr>
<tr>
<td>( \Gamma_W ) [GeV]</td>
<td>2.08849 ± 0.00079</td>
<td>±0.00048</td>
<td>±0.00047</td>
<td>±0.00021</td>
</tr>
<tr>
<td>( \Gamma_Z ) [GeV]</td>
<td>2.49403 ± 0.00073</td>
<td>±0.00059</td>
<td>±0.00031</td>
<td>±0.00021</td>
</tr>
<tr>
<td>( \sigma_h^0 ) [nb]</td>
<td>41.4910 ± 0.0062</td>
<td>±0.0059</td>
<td>±0.0005</td>
<td>±0.0020</td>
</tr>
<tr>
<td>( \sin^2 \theta_{\text{eff}}^{\text{lept}} )</td>
<td>0.23148 ± 0.00012</td>
<td>±0.00000</td>
<td>±0.00012</td>
<td>±0.00002</td>
</tr>
<tr>
<td>( P_T^{\text{pol}} = A_\ell )</td>
<td>0.14731 ± 0.00093</td>
<td>±0.00003</td>
<td>±0.00091</td>
<td>±0.00012</td>
</tr>
<tr>
<td>( A_c )</td>
<td>0.66802 ± 0.00041</td>
<td>±0.00001</td>
<td>±0.00040</td>
<td>±0.00005</td>
</tr>
<tr>
<td>( A_b )</td>
<td>0.934643 ± 0.000076</td>
<td>±0.000003</td>
<td>±0.000075</td>
<td>±0.000010</td>
</tr>
<tr>
<td>( A_{0,\ell} )</td>
<td>0.01627 ± 0.00021</td>
<td>±0.00001</td>
<td>±0.00020</td>
<td>±0.00003</td>
</tr>
<tr>
<td>( A_{0,c} )</td>
<td>0.07381 ± 0.00052</td>
<td>±0.00002</td>
<td>±0.00050</td>
<td>±0.00007</td>
</tr>
<tr>
<td>( A_{0,b} )</td>
<td>0.10326 ± 0.00067</td>
<td>±0.00002</td>
<td>±0.00065</td>
<td>±0.00008</td>
</tr>
<tr>
<td>( R_{\ell}^0 )</td>
<td>20.7478 ± 0.0077</td>
<td>±0.0074</td>
<td>±0.0020</td>
<td>±0.0003</td>
</tr>
<tr>
<td>( R_c^0 )</td>
<td>0.172222 ± 0.000026</td>
<td>±0.000023</td>
<td>±0.000007</td>
<td>±0.000001</td>
</tr>
<tr>
<td>( R_b^0 )</td>
<td>0.215800 ± 0.000030</td>
<td>±0.000013</td>
<td>±0.000004</td>
<td>±0.000000</td>
</tr>
</tbody>
</table>

(negligible uncertainty from \( m_h \) variations)

de Blas et al, 2016
The state of the art on 2 most precisely known quantities

\[ M_W, \quad \sin^2 \theta_{\text{eff}}^l \equiv \frac{1}{4} \left( 1 - \frac{g_V^l}{g_A^l} \right) \]

<table>
<thead>
<tr>
<th>( \delta M_W/\text{MeV} )</th>
<th>( \delta \sin^2 \theta_{\text{eff}}^l/10^{-5} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>higher orders</td>
<td>5</td>
</tr>
<tr>
<td>parametric</td>
<td>9</td>
</tr>
<tr>
<td>exp. current</td>
<td>15</td>
</tr>
<tr>
<td>exp. FCC-ee</td>
<td>0.5</td>
</tr>
</tbody>
</table>

“parametric”: \( \Delta m_t = 1 \, \text{GeV}, \quad \Delta \alpha_{\text{had}}^{(5)} = 3.3 \cdot 10^{-4}, \quad \Delta \alpha_S(M_Z) = 7 \cdot 10^{-4} \)

Degrassi, Gambino, Giardino 2014
general current fit

de Blas et al, 2016

\[ \delta \epsilon_i = \epsilon_i - \epsilon_i^{SM} \]

\[ \epsilon_1^{SM} = 5.21 \cdot 10^{-3}, \quad \epsilon_3^{SM} = 5.28 \cdot 10^{-3} \]

SM EW loops seen with about 20% precision
Two other complementary directions in (the use of) precision data

1. The SM as an effective low-energy theory

\[ \mathcal{L}_{\text{eff}}(E < \Lambda) = \mathcal{L}_{\text{SM}} + \sum_{i,p>0} \frac{c_{i,p}}{\Lambda^p} \mathcal{O}^{(4+p)}_i \]

2. Precision in Higgs couplings

- The slope of the line is the only parameter (\(v\))

(not only ElectroWeak)
EW precision with effective operators

\[ \mathcal{L}_{\text{eff}}(E < \Lambda) = \mathcal{L}_{\text{SM}} + \sum_{i,p > 0} \frac{c_{i,p}}{\Lambda^p} \mathcal{O}_i^{(4+p)} \]

95% lower bounds on \( \Lambda / \text{TeV} \) on one operator at a time

<table>
<thead>
<tr>
<th>( c_i = -1 )</th>
<th>( c_1 = +1 )</th>
<th>( c_i = -1 )</th>
<th>( c_i = +1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((H^+ \tau^a H)W^a_{\mu\nu}B_{\mu\nu})</td>
<td>9.7</td>
<td>10</td>
<td>11.1</td>
</tr>
<tr>
<td>(</td>
<td>H^+ D_\mu H</td>
<td>^2)</td>
<td>4.6</td>
</tr>
<tr>
<td>(i(H^+ D_\mu \tau^a H)(\bar{L}\gamma_\mu \tau^a L))</td>
<td>8.4</td>
<td>8.8</td>
<td>9.8</td>
</tr>
<tr>
<td>(i(H^+ D_\mu \tau^a H)(\bar{Q}\gamma_\mu \tau^a Q))</td>
<td>6.6</td>
<td>6.8</td>
<td>9.6</td>
</tr>
<tr>
<td>(i(H^+ D_\mu H)(\bar{L}\gamma_\mu L))</td>
<td>7.3</td>
<td>9.2</td>
<td>14.8</td>
</tr>
</tbody>
</table>

B, Strumia 2000

deBlas et al 2014

caveats:
In general many more operators already at dim=6
Correlations lost
What is the “true” meaning of these bounds?
Precision in Higgs couplings

\[ \mu_i^f = \frac{\sigma_i \cdot BR_i^f}{(\sigma_i)_{SM} \cdot (BR_i^f)_{SM}} \]

\[ \kappa_f = \frac{g_{h_i f_i} f_i}{(g_{h_i f_i} f_i)_{SM}} \]

\[ \kappa_V = \frac{g_{hVV}}{(g_{hVV})_{SM}} \]
Comparing Higgs with EW precision

Consider any theory where the hVV-coupling $\kappa_V$ deviates from the SM

$$\delta \epsilon_1 = -\frac{3\alpha}{8\pi c^2} (1 - k_V^2) \log \frac{\Lambda}{m_h}, \quad \delta \epsilon_3 = \frac{\alpha}{24\pi s^2} (1 - k_V^2) \log \frac{\Lambda}{m_h}$$

B, Bellazzini, Rychkov, Varagnolo 2007

EW precision in principle more constraining on $\kappa_V$ however:

Need to specify the cutoff and be sure of no other contribution
The single prediction of the SM in quark flavour physics

\[ J^\mu_W|_{\text{quarks}} = \bar{u}_L \gamma^\mu d_L \]

the only FV interaction with

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \]
A significant comparison

\[ \epsilon_1^{SM} = 5.21 \cdot 10^{-3}, \quad \epsilon_3^{SM} = 5.28 \cdot 10^{-3} \]

measures EW loops at about 20% level

A future facility (FCCee, ...) could go to 2% level

B, Buttazzo, Sala, Straub 2014

An "aggressive" flavour program could go to 2% level
An alternative definition of the SM (equally precise!)

1. Symmetry group $\mathcal{L} \times \mathcal{G}$

$\mathcal{L} = $ Lorentz (rigid, exact)
$\mathcal{G} = SU(3) \times SU(2) \times U(1)$ (local, spontaneously broken)

2. Particle content (rep.s of $\mathcal{L} \times \mathcal{G}$) – See below

3. All “operators” (products of $\Phi, \partial_\mu \Phi$) in $\mathcal{L}$ of dimension $\leq 4$ with a single exception

$\theta G_{\mu \nu} \tilde{G}^{\mu \nu}$

$h = c = 1 \Rightarrow [A_\mu] = [\phi] = [\partial_\mu] = M, \quad [\Psi] = M^{3/2}, \quad [\mathcal{L}] = M^4$
The particles of the Standard Model (SM)

\[ J = 0 \]

\[ \Psi_i = \]

\[ J = 1/2 \]

\[ J = 1 \]

<table>
<thead>
<tr>
<th>( J = 0 )</th>
<th>( J = 1/2 )</th>
<th>( J = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u(1968) )</td>
<td>( c(1974) )</td>
<td>( t(1994)^* )</td>
</tr>
<tr>
<td>( d(1968) )</td>
<td>( s(1968) )</td>
<td>( b(1977) )</td>
</tr>
<tr>
<td>( e(1897) )</td>
<td>( \mu(1937) )</td>
<td>( \tau(1975) )</td>
</tr>
<tr>
<td>( \nu_e(1956) )</td>
<td>( \nu_\mu(1962) )</td>
<td>( \nu_\tau(2000)^* )</td>
</tr>
</tbody>
</table>

\[ i = \]

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h(2012) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

H = \((pe)\)  \( p = (uud) \)  \( n = (udd) \)

\( G_\mu^a(1978)^* \)  \( A_\mu(1905) \)  \( W_\mu(1984) \)  \( Z_\mu(1984) \)

A complete story?  A single scalar?
Representation content and accidental symmetries

\[ \Psi = Q(3, 2)_{1/6} \ u(\bar{3}, 1)_{-2/3} \ d(\bar{3}, 1)_{1/3} \ L(1, 2)_{-1/2} \ e(1, 1)_1 \]

(An important hint for “algebraic” Unification?)

From \[ O_i : d(O_i) \leq 4 \]

\[ \Rightarrow B, \ L_e, L_\mu, L_\tau \]

and \[ U(3)^3 \equiv U(3)_Q \times U(3)_u \times U(3)_d \text{ only broken by } Y_u, Y_d \]
An interesting story about symmetries

∞'s ⇒ renormalizable th.s $O_d(\Phi, \partial_\mu \Phi)$ $d \leq 4$ ⇒ $d > 4$

30's

40's - 50's

70's

↓

Accidental symmetries (approximate)

Parity in the electromagnetic interactions

Isospin, $SU(3)$, chiral symmetry in strong interactions

Barion (B) and Lepton ($L_i$) numbers in the full SM

\[
B = N_q - N_{\bar{q}} \\
L_i = N_{l_i} - N_{\bar{l_i}}
\]

$p \rightarrow e^+ + \pi^0$
Lepton Flavour Violation is absent in the SM

- SINDRUM II
  - $B(\mu \to e\gamma) < 4.3 \times 10^{-12}$
  - $B(\mu \to e\gamma) < 7 \times 10^{-13}$

- 2006
  - $\mu \to e\gamma$

- 2012
  - B-factories
  - $3.3 \div 4.5 \times 10^{-8}$

- MEG@PSI
  - $5.7 \times 10^{-13}$

- SINDRUM
  - $1 \times 10^{-12}$

- 1988
  - $\mu \to e\gamma$

An aside story

- BNL E821
  - $(g - 2)_\mu$
  - $(296 \pm 81) \times 10^{-11}$

- 2004
  - A hint for NP?
Conclusions postponed to end of lecture II
For question time
vacuum stability

\[ V(\varphi) = \mu^2 |\varphi|^2 + \lambda |\varphi|^4 \]

\[ \frac{d\lambda}{d \log Q} = \frac{3}{2\pi^2} \left[ \lambda^2 + \frac{1}{2} \lambda y_t^2 - \frac{1}{4} y_t^4 + \cdots \right] \]

With current values of \( m_H, m_t, \alpha_S, \ldots \)

\[ \lambda(\approx 10^{11} \text{ GeV}) < 0 \]

\[ m_W = gv/\sqrt{2} \]
\[ m_H = 2\sqrt{\lambda v} \]
\[ m_t = y_tv \]

\[ \Rightarrow \text{A second minimum of } V \text{ at } \phi \gtrsim 10^{11} \text{ GeV} \]
\[ \text{to which } \nu \text{ should tunnel in a very long time (} \gg t_{Univ} \text{)} \]

- Is there a real meta-stability at \( \phi < M_{Pl} \) ?
- Any experimental implication?
- Connection to inflation?
- Is it a problem?
Landau poles

\[ \frac{dg_1^2}{dt} = \frac{41}{40} g_1^4 \Rightarrow \text{a Landau pole at } \Lambda_1 \]

- the problem not cured by including other couplings
- can it be cured by gravity? Yes, since \( \Lambda_1 > M_{Pl} \), if gravity important at \( E \lesssim M_{Pl} \)
- what if gravity softened enough, so that it becomes irrelevant? (How is hard to tell, but...)
- need \( SU(3) \times SU(2) \times U(1) \) fully immersed in a non-abelian group

\[
SU(4)_{PS} \times SU(2)_L \times SU(2)_R
\]
\[
SU(3)_c \times SU(3)_L \times SU(3)_R
\]

which requires heavier scales than \( \nu \)
$(\sum m_{\nu})$ determination

(a recent result from KamLAND)

$m_{\beta\beta} < 0.06 \div 0.16 \text{ eV}$

Dell’Oro et al 2015

Palanque-Delabrouille et al 2015