II. Problems of (questions for) the Standard Model

R. Barbieri
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Problems of (questions for) the SM

0. Which rationale for matter quantum numbers?

\[|Q_p + Q_e| < 10^{-21}e\]

1. Phenomena unaccounted for

neutrino masses  
matter-antimatter asymmetry

Dark matter  
inflation?

2. Why \( \theta \lesssim 10^{-10} \)?

Axions

3. \( \mathcal{O}_i : d(\mathcal{O}_i) \leq 4 \) only?

neutrino masses  
Are the protons forever?

Gravity

4. Lack of calculability (a euphemism)

the hierarchy problem  
the flavour paradox
Why \(|Q_p + Q_e| < 10^{-21}e\) ?

(recall Einstein’s lesson from \(m_{in} = m_{grav}\))

\[
\Psi = Q(3, 2)_{1/6} \ u(\bar{3}, 1)_{-2/3} \ d(\bar{3}, 1)_{1/3} \ L(1, 2)_{-1/2} \ e(1, 1)_1
\]

\(\Psi = \text{next-to-simplest rep of } \mathcal{G}:\)
chiral, anomaly-free, vector-like under \(SU(3) \times U(1)_{em}\)

However:

1. A simpler rep:
   \(\Xi = (3, 2)_0 \ (\bar{3}, 1)_{1/2} \ (\bar{3}, 1)_{-1/2}\)

2. What if \(\nu_R\) are added?

\[
\tilde{\Psi} = Q(3, 2)_y \ u(\bar{3}, 1)_{-y-1/2} \ d(\bar{3}, 1)_{-y+1/2} \ L(1, 2)_{-3y} \ e(1, 1)_{5y+1/2} \ \nu^c(1, 1)_{3y-1/2}
\]

(An important hint for “algebraic” Unification?)
The unification way: $SU(5)$

A unique “embedding” of $SU_{3,2,1}$ into $SU_5$

$$
\left( \frac{1}{2} \lambda^i_{3 \times 3} \mid \frac{1}{2} \sigma^a_{2 \times 2} \right) \quad Y \propto \left( \frac{1_{3 \times 3}}{\frac{3}{2} \cdot 1_{2 \times 2}} \right)
$$

The particle content follows in the simplest reps

$$
\bar{5} = \begin{pmatrix}
    d^c_1 \\
    d^c_2 \\
    d^c_3 \\
    e^- \\
    -\nu_e
\end{pmatrix} \quad 10 = \begin{pmatrix}
    0 & u^c_3 & -u^c_2 & -u_1 & -d_1 \\
    u^c_1 & 0 & 0 & -u_2 & -d_2 \\
    u_3 & -d_3 \\
    0 & -e^c & 0
\end{pmatrix}
$$

with all quantum numbers fixed (including hypercharge)
Neutrino masses

Known to be nonzero since about 1990
Yet vanishing in the SM because of an accidental symmetry: L-conservation

\((A, Z) \rightarrow (A, Z + 1) + e + \bar{\nu}\)

Accidental symmetries are not exact

\[\Delta \mathcal{L} = \frac{(LH)(LH)}{M}\]

Neutrinos are massive and of Majorana type \((\nu = \bar{\nu})\)

Should observe: \((A, Z) \rightarrow (A, Z + 2) + 2e\)

So far \(\tau(2\beta^{0\nu}) \gtrsim 10^{25}\) years
Neutrino oscillations
(in the standard 3-neutrino framework)

\[
|\nu_{l\alpha}\rangle = \begin{cases} 
|\nu_\alpha\rangle & t = 0 \\
R = ct & t
\end{cases}
\]

\[
|\nu_i\rangle = V_{i\alpha}|\nu_\alpha\rangle
\]

\[
\alpha = e, \mu, \tau
\]

\[
<\nu_\beta|\nu_\alpha, t> = \sum_{i=1,2,3} V_{i\beta}^* V_{i\alpha} e^{-\frac{m_i^2 t}{2p}}
\]

+ 2 more phases if \(\nu\)'s are Majorana, not affecting oscillations

the absolute scale yet unknown
3 ways to be sensitive to the absolute $\nu$-mass scale

1- beta-decay endpoint

$$m_\beta = \left[ c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2 \right]^{\frac{1}{2}}$$

2- neutrino-less $\beta\beta$-decay

$$m_{\beta\beta} = \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3} \right|$$

3 - cosmology (large scale structures)

$$\Sigma = m_1 + m_2 + m_3$$
Power spectrum of large scale structures

\[ \delta \equiv \frac{\rho(\mathbf{r}) - \bar{\rho}}{\bar{\rho}} \quad \text{Fourier tr} \quad \rightarrow \quad \delta(\mathbf{k}) \]

\[ \xi(\mathbf{r}) \equiv \langle \delta(\mathbf{x} + \mathbf{r})\delta(\mathbf{x}) \rangle \quad \text{Fourier tr} \quad \rightarrow \quad |\delta(\mathbf{k})|^2 \equiv P(\mathbf{k}) \]

the neutrino fluid influences \( P_m(k) \) by gravitational interactions

\[ \frac{P_m(k)_{\Sigma \neq 0}}{P_m(k)_{\Sigma = 0}} \]

\[ \sum = m_1 + m_2 + m_3 \]

Lesgourgues, Pastor 2006
Lesgourgues et al. 2013

Not independent on "priors" but still highly significant

"free streaming"

Power spectrum of large scale structures

Determination with future large-scale structure observations (Euclid) at 2 – 5σ depending on control of (mildly) non-linear physics

Lesgourgues et al. 2013

Not independent on "priors" but still highly significant
current bounds (with uncertainties)

\[ \frac{m_{\beta\beta}}{eV} \]

inverted

normal

\[ \frac{\Sigma}{eV} \]

LSS + CMB 2015

green = optimistic

black = realistic/pessimistic

Kamland 2016
Key neutrino measurements

\( m_\beta \)

beta-decay endpoint

\( m_{\beta\beta} \)

neutrino-less \( \beta\beta \) decay

\[ \Sigma = m_1 + m_2 + m_3 \]

large scale structures

Lisi et al

2\( \sigma \) bounds from current knowledge of oscillations only

hypothetical measurements
2. Why $\theta \lesssim 10^{-10}$?

$\theta G_{\mu\nu} \tilde{G}^{\mu\nu}$

How do we know that $\theta \lesssim 10^{-10}$?

$\theta G_{\mu\nu} \tilde{G}^{\mu\nu}$ is T-odd and (almost) the only source of T-violation in the SM

<table>
<thead>
<tr>
<th>$\tilde{\mu} \cdot \vec{B}$</th>
<th>$\vec{d} \cdot \vec{E}$</th>
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<tbody>
<tr>
<td>$T$</td>
<td>$+$</td>
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<td>$-</td>
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$|\mu_N^-| = 2 \cdot 10^{-14} \text{e} \cdot \text{cm}$

$|d_N^-| \approx \theta \cdot 10^{-15} \text{e} \cdot \text{cm}$

$|d_N^-|_{\text{exp}} < 3 \cdot 10^{-26} \text{e} \cdot \text{cm}$

$\Rightarrow$ Make $\theta$ a dynamical field forced in its cosmological history to relax to 0 (almost) and (possibly) appear as DM
A quick introduction to axions

2 field theory results that you should know:

1. In spite of being a 4-divergence \( \mathcal{L}_\theta = \theta \ G_{\mu \nu}^a \tilde{G}^{\mu \nu}_a \) is physical

   In a non-abelian case, there are pure gauge configurations that give a non-vanishing contribution to \( S[A^a_\mu] \) at infinity
   Crucial to solve the "\( \eta \)" problem in QCD

2. Due to the triangle anomaly

   \[ J_{\mu 5} = \bar{q} \gamma_\mu \gamma_5 q \]

   \[ \partial_\mu J_{\mu 5} = \frac{\alpha_S N}{8\pi} G_{\mu \nu}^a \tilde{G}^{\mu \nu}_a \]

   In fact, by a chiral transformation that makes \( M_q \) physical in

   \[ \mathcal{L}_M = \bar{q}_R M_q q + h.c. \]

   \[ \int d^4x \mathcal{L} \rightarrow \text{Arg det} M_q \int d^4x \partial_\mu J_{\mu 5} \] so that

   \[ \theta_{\text{eff}} = \theta + \text{Arg det} M_q \] is the physical combination

   (out of \( U(N)_L \times U(N)_R \) only \( U(1)_A \) anomalous)
A quick introduction to axions

To solve the strong CP problem:

Embed the chiral symmetry into an exact classical $U(1)$-symmetry (PQ) spontaneously broken at a scale $f_a$

Classical examples:

**DFS**

$$\mathcal{L} = \lambda S H_u H_d + Y_u \bar{Q} H_u u + Y_d \bar{Q} H_d d + Y_e \bar{Q} H_d e$$

**KSVZ**

$$\mathcal{L} = \lambda S \bar{T} T + \bar{T} \gamma^\mu D_\mu T$$  with $T$ a new QCD triplet

The axion $a(x)$ is the corresponding (pseudo)GB
A simplified laboratory

Consider a gauged $U(1)_A$

<table>
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<th>$f_L$</th>
<th>$f_R$</th>
<th>$\phi$</th>
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<tbody>
<tr>
<td>$Q$</td>
<td>1</td>
<td>0</td>
<td>$-1$</td>
</tr>
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</table>

\[ \mathcal{L}_Y = g_Y \phi \bar{f}_L f_R + h.c. \]

\[ \phi = (\nu + h)e^{i \frac{a}{\nu}} \]

\[ J_\mu = \bar{f}_L \gamma_\mu f_L - \nu \partial_\mu a \]

Naively
\[ \partial_\mu J_\mu = -ig_Y \nu \bar{f} \gamma_5 f - \nu \partial_\mu^2 a = 0 \]

However, because of the anomaly, under a gauge transformation
\[ a \to a + \nu \epsilon \quad \delta \mathcal{L} = \epsilon \partial_\mu J_\mu = \frac{g^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \neq 0 \]

unless one adds to the Lagrangian
\[ \Delta \mathcal{L} = \frac{a}{\nu} \frac{g^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \]

so that
\[ \delta (\mathcal{L} + \Delta \mathcal{L}) = 0 \]
The axion Lagrangian

\[ \mathcal{L}_a = -\frac{1}{2} |\partial_\mu a|^2 + \frac{\partial_\mu a}{f_a} J_{\mu PQ} + \frac{a}{f_a} \frac{\alpha_S}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} \]

to keep the formal \( U(1)_{PQ} \) invariance: \( a \rightarrow a + \alpha f_a \)

Useful to make the transformation to get rid of \( a \)

\[ q = \begin{pmatrix} u \\ d \end{pmatrix} \rightarrow e^{i\gamma_5 \frac{a}{f_a} Q} q \]

\[ Q_a = \frac{1}{2} \mathbb{1} \]

\[ \mathcal{L}_a \rightarrow \frac{\partial_\mu a}{f_a} (J_{\mu PQ} - \bar{q} \gamma_\mu \gamma_5 Q a q) - (\bar{q}_L \tilde{M}_q(a) q_R + h.c.) \]

\[ \tilde{M}_q(a) = e^{i \frac{a}{f_a} Q} M_q e^{i \frac{a}{f_a} Q} \]

\[ \langle \bar{q}_L q_R \rangle = B f_\pi^2 e^{i\Pi/f_\pi} \]

\[ \langle a \rangle = 0 \quad \Rightarrow \text{no CPV} \]

\[ m_a^2 = \frac{m_u m_d}{m_u + m_d} \frac{m_\pi^2 f_\pi^2}{f_a^2} \]
Relic abundance of the QCD axion

\[ 3H \approx m_a \]

\[ H = T^2 / M_{Pl} \]

\[ T > \Lambda_{QCD} \quad T < \Lambda_{QCD} \]

\[ \frac{m_a^2}{f_a} \left( \frac{\Lambda_{QCD}}{T} \right)^4 \]

\[ \frac{m_a^2}{f_a} \]

\[ \rho_a = m_a^2 a^2 \propto T^3 \propto 1/R^3 \]

\[ \text{i.e. cold Dark Matter} \]

\[ \ddot{a} + 3H \dot{a} + m_a^2 a = 0 \]
QCD Axions in cosmology

\[ m_a f_a \approx 10^{-4} \text{ eV} \cdot 10^{11} \text{ GeV} \]

\[ \Omega_a h^2 \approx 0.16 \left( \frac{m_a}{10^{-5} \text{ eV}} \right)^{-1.18} \theta_i^2 \]

\[ \theta_i = \frac{a_i}{f_a} \]

\[ \theta_i^2 = \frac{\pi^2}{3} \]

(Axion Like Particles: \( m \) and \( f \) unrelated)
The dynamical field, $a$, is the “axion” and is very intensively searched for (with the most interesting region still unaccessible).
The “hierarchy” problem
Can we calculate the Higgs mass? NOT in the SM

If we try: \[ V(h) = m^2(\alpha, \beta)|h|^2 + \lambda|h|^4 \]

To get \(<h> = 175\,\text{GeV}\), as observed, we have to live very very close to the critical line

But we don’t have knobs!
The Higgs naturalness problem illustrated in another way

Take the SM + a particle of mass $M_H = 10^{10} \text{ GeV}$ and coupling $\lambda_H$ to the Higgs boson

The running $m_h^2$ versus the scale $M$

$m_h = 125 \text{ GeV}$

$\propto \log(M)$

A jump at $M_H$ of size

$$\frac{(\lambda_H M_H)^2}{16\pi^2}$$

$m_h$ depends on a very precise initial condition of order $O(m_h^2/m_H^2)$ at some short distance

“fine tuning”
The hierarchy problem, once again

Can we compute the Higgs mass/vev in terms of some fundamental dynamics?

NOT in the SM

\[ \delta m_h^2 = \frac{3y_t^2}{4\pi^2} \Lambda_t^2 - \frac{9g^2}{32\pi^2} \Lambda_g^2 - \frac{3g'^2}{32\pi^2} \Lambda_g'^2 + \ldots \]

\[ \Lambda_t \lesssim 0.4\sqrt{\Delta} \text{ TeV} \quad \Lambda_g \lesssim 1.1\sqrt{\Delta} \text{ TeV} \quad \Lambda_g' \lesssim 3.7\sqrt{\Delta} \text{ TeV} \]

\[ 1/\Delta = \text{amount of tuning} \]

⇒ Look for a top “partner” (coloured, S=0 or 1/2) with a mass not far from 1 TeV
The flavour paradox

Yukawa couplings: a piece of physical reality

as opposed to:

?!?!?
Summary of lectures I and II

The Standard Model is **NOT** a complete story

Pictures that go **Beyond the SM** are not lacking, but - fair to say - we don’t know which one is right

The very nature of Particle Physics and the current uncertain situation **REQUIRE** highly diverse frontiers of research

Can an understanding of short distance physics ever be produced deeper than the SM one?

Could such a putative theory not include the SM as a relevant limit?
The SM as an emerging iceberg

What there is under the water?
BSM in the multi TeV region...
BSM in the multi TeV region...

... or the SM extended up to $E \gg \text{TeVs}$?
For question time
vacuum stability

\[ V(\varphi) = \mu^2|\varphi|^2 + \lambda|\varphi|^4 \]

\[ \frac{d\lambda}{d\log Q} = \frac{3}{2\pi^2} \left[ \lambda^2 + \frac{1}{2}\lambda y_t^2 - \frac{1}{4}y_t^4 + \cdots \right] \]

With current values of \( m_H, m_t, \alpha_S, \ldots \)

\[ \lambda(\approx 10^{11}\text{ GeV}) < 0 \]

⇒ A second minimum of \( V \) at \( \phi \gtrsim 10^{11}\text{ GeV} \)

to which \( \nu \) should tunnel in a very long time (\( \gg t_{\text{Univ}} \))

- Is there a real meta-stability at \( \phi < M_{Pl} \) ?
- Any experimental implication?
- Connection to inflation?
- Is it a problem?
Landau poles

\[
\frac{d}{dt} g = \frac{1}{4N_c} g^4
\]

in a non-abelian group

in a non-abelian group

\[
\text{SU}(3) \times \text{SU}(2) \times \text{SU}(2) \times \text{SU}(2) \times \text{SU}(3)
\]

fully immersed

fully immersed

- need

- need

- need

the problem not cured by including other couplings

Landau poles

\[
V_I \quad \text{a Landau pole at} \quad V_I = \frac{16V_V}{1V} = \frac{\mu p}{16p}
\]

Landau poles

\[
\ld W \gtrsim E
\]

\[
\ld W < V_I
\]

\[
\text{if gravity important at}
\]

\[
\text{can it be cured by gravity? Yes, since}
\]

\[
\text{what if gravity softened enough, so that it becomes}
\]

\[
\text{irrelevant? (How is hard to tell, but...)}
\]

\[
\text{irrelevant? (How is hard to tell, but...)}
\]
LHC now

Not a serious problem at a fundamental level

Things do not work the way they were originally thought

An indicative MSSM

Some NMSSM

Hard to achieve

Model dependent

How dramatic is the "little hierarchy problem"?
A self-critical Higgs vev

1. A Goldstone boson of a U(1) broken at a scale $f$

2. A U(1)-breaking coupling of $H$ to $S$

3. A breaking of $U(1)$ controlled by a small mass parameter entering the Higgs mass term $m$

\[ V = \frac{f}{\phi} \bar{\epsilon} S = S \]

\[ \phi \bar{V}w + \bar{v} |H| \bar{\chi} + \bar{\tau} |H| (\phi w - \bar{z} V) + \frac{f}{+S + S} (H) d + \bar{v} |S| + \bar{z} |S| \bar{z} f - = \Lambda \]

A. A Goldstone boson of a U(1) broken at a scale $f$

B. A U(1)-breaking coupling of $H$ to $\phi$

C. A breaking of $U(1)$ controlled by a small mass parameter $m$ entering the Higgs mass term
\[
\frac{\partial \omega}{\partial \psi V} \gtrsim \frac{w}{\varphi V} \approx \phi \\
\frac{\frac{\partial V}{\partial \psi \omega}}{\frac{\partial V}{\partial \omega}} \gtrsim \frac{\frac{\partial V}{\partial \omega}}{\frac{\partial \omega}{\partial \omega}} = w
\]

Changes by \( O(1) \)

By \( O(1) \)

Natural \( \omega = \psi \)

\[
\frac{1}{\varphi w - \varphi V} \omega \approx \psi \iff 0 = \frac{\phi \varphi}{\Lambda \varphi} \\
0 < \frac{\chi}{\varphi w - \varphi V} \approx \varphi \psi \iff 0 = \frac{\psi \varphi}{\Lambda \varphi}
\]

(non trivial)

\[
\begin{aligned}
\varphi \frac{\partial \omega}{\partial \psi} < \frac{\partial \omega}{\partial \psi} & \quad \ldots \quad + \varphi \frac{\partial \omega}{\partial \omega} \frac{\partial \omega}{\partial \omega} + \varphi \frac{\partial \omega}{\partial \omega} + \varphi \omega = (H)\omega \\
\phi \psi V \omega + \psi \psi H \psi + \varphi \psi H (\phi w - \varphi V) + f / \phi \cos (H) d = \Lambda
\end{aligned}
\]

\( (\phi, H) \Lambda \) \( \text{Minimizing} \)
Experimental consequences?

Rolling stops when barriers grow due to \( 0 < \alpha \). Until it hits value where \( m^2 \) cross zero slow-rolls during inflation at \( 0 = \alpha \). Historically, evolution of \( \phi \) (and of \( \Lambda \)).

(under suitable conditions: e.g. a very very long inflation period)
\( \frac{\mathcal{M}_3}{\mathcal{M}_3^0} > 0.06 \div 0.16 \mathcal{E} \quad (\text{a recent result from KamLAND}) \)