The Standard Model and (some of) its extensions

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IV. Anomalies in B-decays
A deviation from the SM in flavour, finally?

\[ R(D^*) = \frac{\mathcal{B}(B \rightarrow D^{(*)}\tau\nu)}{\mathcal{B}(\rightarrow D^{(*)}l\nu)} = 1.27 \pm 0.06 \]

\[ l = \mu, e \]
A deviation from the SM in flavour, finally?

\[ R(D^*) = \frac{\mathcal{B}(B \rightarrow D^*\tau\nu)}{\mathcal{B}(\rightarrow D^*l\nu)} \]

A 4\sigma deviation from the SM from a collection of different (difficult) experiments (no problem from the theory error)
Which direction to take?

1. High energy exploration

\[ \mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{C_i^\alpha}{\Lambda_i^\alpha} (\bar{f} f \bar{f} f)^\alpha_i \]

\[ \alpha = K(\Delta S = 2), \quad D(\Delta C = 2), \quad B_d(\Delta B = 1), \quad B_s(\Delta B = 1) \]

\[ i = 1, \ldots, 5 = \text{different Lorentz structures} \]

2. Indirect signals of new physics at the TeV scale
Minimal Flavour Violation in the quark sector

**Phenomenological Definition:**

In EFT the only relevant op.s correspond to the FCNC loops of the SM, weighted by a single scale $\Lambda$ and by the standard CKM factors (up to $O(1)$ coeff.s)

**Strong MFV**

$$U(3)_Q \times U(3)_u \times U(3)_d$$

$$Y_u = (3, \bar{3}, 1) \rightarrow Y_u^D$$

$$Y_d = (3, 1, \bar{3}) \rightarrow VY_d^D$$

$$A(d_i \rightarrow d_j) = V_{tj}V_{ti}^* A_{SM}^{\Delta F=1}(1 + a_1\left(\frac{4\pi M_W}{\Lambda}\right)^2)$$

$$M_{ij} = (V_{tj}V_{ti}^*)^2 A_{SM}^{\Delta F=2}(1 + a_2\left(\frac{4\pi M_W}{\Lambda}\right)^2)$$

Chivukula, Georgi 1987
Hall, Randall 1990
D’Ambrosio et al 2002
Weak MFV

\[ U(2)_Q \times U(2)_u \times U(2)_d \times U(1)_{d3} \]

\[ y_b = (1, 1, 1)_{-1} \quad \lambda_u = (2, \bar{2}, 1)_0 \quad \lambda_d = (2, 1, \bar{2})_0 \quad V_Q = (2, 1, 1)_0 \]

1. gives a symmetry status to heavy and weakly mixed top

2. only broken by small spurions \((\lesssim 4 \cdot 10^{-2})\)

\[
\Rightarrow \quad Y_u = \begin{pmatrix} \frac{\lambda_u}{0} & y_t x_t V \\ 0 & \frac{-\lambda_u y_t y_t}{y_t} \end{pmatrix} \quad Y_d = \begin{pmatrix} \frac{\lambda_d}{0} & y_b x_b V \\ 0 & \frac{-\lambda_d y_b y_b}{y_b} \end{pmatrix} \quad v = (0) \]

mimicked in the lepton sector by: \( U(2)_L \times U(2)_e \times U(1)_{e3} \)

\[ y_\tau = (1, 1)_{-1} \quad \lambda_e = (2, \bar{2})_0 \quad V_L = (2, 1)_0 \]

(except for neutrinos, due to \( N^T_R \cdot MN_R \))
B-physics “anomalies”

1. $b \rightarrow c \tau \nu$

$$R_{D^*}^{\tau/\ell} = \frac{B(B \rightarrow D^* \tau \nu)_{\exp}}{B(B \rightarrow D^* \ell \nu)_{\exp}} / \frac{B(B \rightarrow D^* \tau \nu)_{SM}}{B(B \rightarrow D^* \ell \nu)_{SM}} = 1.28 \pm 0.08$$

$$R_D^{\tau/\ell} = \frac{B(B \rightarrow D \tau \nu)_{\exp}}{B(B \rightarrow D \ell \nu)_{\exp}} / \frac{B(B \rightarrow D \tau \nu)_{SM}}{B(B \rightarrow D \ell \nu)_{SM}} = 1.37 \pm 0.18,$$

2. $b \rightarrow s l^+ l^-$

$$R_K^{\mu/e} = \left. \frac{B(B \rightarrow K \mu^+ \mu^-)_{\exp}}{B(B \rightarrow K e^+ e^-)_{\exp}} \right|_{q^2 \in [1,6] \text{GeV}} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

(could be related to the $P'_5$ anomaly in the $q^2$ distribution)

Both a $20 \div 30\%$ deviation from the SM

However tree (1) versus loop level (2)!
Question

Is there a flavour group $G_F$ and a tree level exchange $\Phi$ such that:

1. With unbroken $G_F$, $\Phi$ couples to the third generation of quarks and leptons only;

2. After small $G_F$ breaking, the needed operators are generated

$$\left(\bar{c}_L\gamma_\mu b_L\right)\left(\bar{\tau}_L\gamma_\mu \nu_L\right)$$

$$\left(\bar{b}_L\gamma_\mu s_L\right)\left(\bar{\mu}\gamma_\mu \mu\right) \text{ at suppressed level}$$
Answer

\[ G_F = G_F^q \times G_F^l \quad \text{“minimally” broken} \]

\[ G_F^q = U(2)_Q \times U(2)_u \times U(2)_d \times U(1)_{d3} \]

\[ G_F^l = U(2)_L \times U(2)_e \times U(1)_{e3} \]

with mediators under \( SU(3) \times SU(2) \times U(1) \):

1. \( V_\mu = (3, 1)_{2/3} \) Lorentz vector, \( G_F \) singlet
2. \( V_\mu = (3, 3)_{2/3} \) Lorentz vector, \( G_F \) singlet
3. \( \Phi = (3, 3)_{-1/3} \) Lorentz scalar, \( G_F \) singlet
Couplings in the physical bases

\[ \mathcal{L}_1 = g_U (\bar{u}_L \gamma^\mu F^U \nu_L + \bar{d}_L \gamma^\mu F^D e_L) U_\mu + \text{h.c} \]

and similar for \( \mathcal{L}_{2,3} \)

\[
F^U = \begin{pmatrix}
-V_{ub} (s_l \epsilon_l) [1 - r_d] & -V_{ub} (c_l \epsilon_l) [1 - r_d] & V_{ub} [1 - r_d] \\
-V_{cb} (s_l \epsilon_l) [1 - r_d] & -V_{cb} (c_l \epsilon_l) [1 - r_d] & V_{cb} [1 - r_d] \\
-V_{tb} (s_l \epsilon_l) & -V_{tb} (c_l \epsilon_l) & V_{tb}
\end{pmatrix}
\]

\[
F^D = \begin{pmatrix}
-V_{td} (s_l \epsilon_l) r_d & -V_{td} (c_l \epsilon_l) r_d & V_{td} r_d \\
-V_{ts} (s_l \epsilon_l) r_d & -V_{ts} (c_l \epsilon_l) r_d & V_{ts} r_d \\
-V_{tb} (s_l \epsilon_l) & -V_{tb} (c_l \epsilon_l) & V_{tb}
\end{pmatrix}
\]

in terms of \( r_d, \epsilon_l, \theta_l \)
\[ \mathcal{L}_2 = \frac{g_\tilde{U}}{\sqrt{2}} \left[ \frac{1}{\sqrt{2}} (\bar{u}_L \gamma^\mu F^U \nu_L - \bar{d}_L \gamma^\mu F^D e_L) U_\mu^{2/3} + (\bar{u}_L \gamma^\mu F^U e_L) U_\mu^{5/3} + (\bar{d}_L \gamma^\mu F^D \nu_L) U_\mu^{-1/3} \right] + \text{h.c} \]

\[ \mathcal{L}_3 = \frac{g_\tilde{S}}{\sqrt{2}} \left[ \frac{1}{\sqrt{2}} (\bar{u}_L^c F^U e_L + \bar{d}_L^c F^D \nu_L) S^{1/3} + (\bar{u}_L^c F^U \nu_L) S^{-2/3} + (\bar{d}_L^c F^D e_L) S^{4/3} \right] + \text{h.c} \]
Tree level effects

In terms of

\[ (R_U, R_{\bar{U}}, R_{\bar{S}}) \equiv \frac{4M_W^2}{g^2} \left( \frac{g_U^2}{M_U^2}, \frac{g_{\bar{U}}^2}{M_{\bar{U}}^2}, \frac{g_{\bar{S}}^2}{M_{\bar{S}}^2} \right) \]

\[ b \rightarrow c \tau \nu \]

\[ R_{D(*)}^{\tau/l} \approx 1 + (R_U, -\frac{1}{4} R_{\bar{U}}, -\frac{1}{8} R_{\bar{S}})(1 - r_d) \]

\[ b \rightarrow s \nu \bar{\nu} \]

\[ R_{K(*)\nu} = \frac{\mathcal{B}(\bar{B} \rightarrow K(*)\nu \bar{\nu})}{\mathcal{B}(\bar{B} \rightarrow K(*)\nu \bar{\nu})_{SM}} \approx \frac{1}{3} \left( 3 + 2\text{Re}(x) + |x|^2 \right) \]

\[ (x_U, x_{\bar{U}}, x_{\bar{S}}) = -\frac{\pi}{\alpha c_{\nu}^{SM}} r_d \left( 0, -\frac{R_{\bar{U}}}{2}, \frac{R_{\bar{S}}}{8} \right) \]
$b \rightarrow c\tau \nu$

$S$

$\beta = 1 - r_S (1 - a)$

$R_S$

$\Rightarrow$ Only $\mathcal{U}_\mu$ survives tree level test (trivially)

but can one make sense of a vector lepto-quark?
Interpreting the lepto-quark as a $\rho$-like state in a composite Higgs picture

The global group $\mathcal{G}$ of a strong dynamics is broken down to a subgroup $\mathcal{H}$, producing a (pseudo)Goldstone boson $H$

In the “standard” picture:

$$\mathcal{G} = SU(3) \times SO(5) \times U(1)_X \xrightarrow{f} \mathcal{H} = SU(3) \times SO(4) \times U(1)_X$$

Extend it to:

$$\mathcal{G} = SU(4) \times SO(5) \times U(1)_X \xrightarrow{f} \mathcal{H} = SU(4) \times SO(4) \times U(1)_X$$
General structure of composite Higgs models

(a general insertion)

The global group $G$ of a strong dynamics is broken down to a subgroup $H$, producing a (pseudo)Goldstone boson $H$

common ingredients:

- composite vectors $\hat{V}_\mu$ in the adjoint of $H$
- composite fermions $F$ in full reps of $H$
- elementary vectors $V_\mu$ in the adjoint of $SU_{321} \subset H$
- elementary fermions $f$ in the standard reps under $SU_{321} \subset H$
- mass mixings $\bar{F}f$ and $\hat{V}_\mu V_\mu$ consistent with $SU_{321} \subset H$
An explicit example ("2 site")

\[ \mathcal{H} = U(1)_{T_{3R}} \times U(1)_{X} \quad g_{el} = U(1)_{Y} \quad Y = T_{3R} + X \]

1. Double \[ \mathcal{H} \Rightarrow U(1)_{T_{3R}}^{el} \times U(1)_{X}^{el} \times U(1)_{T_{3R}} \times U(1)_{X} \]

2. Gauge \[ g^{gauge} = U(1)_{Y}^{el} \times U(1)_{T_{3R}} \times U(1)_{X} \]

3. Introduce \[ \Sigma_{T} = (1/2, 0, -1/2, 0) \text{ and } \Sigma_{X} = (0, 1/2, 0, -1/2) \]
with \[ < \Sigma_{T} > = f_{T} \text{ and } < \Sigma_{X} > = f_{X} \]

4. Under \[ g^{gauge} \]
   "elementary" fermions \[ f = (1, 0, 0) \]
   "composite" fermions \[ F = (0, 1/2, 1/2) \]
An explicit example ("2 site")

\[ \mathcal{H} = U(1)_{T_{3R}} \times U(1)_X \quad \mathcal{G}_{el} = U(1)_Y \quad Y = T_{3R} + X \]

\[ M_V^2 = \begin{pmatrix} \hat{g}^2(f_T^2 + f_X^2) & -\hat{g}\hat{g}_X f_X^2 & -\hat{g}\hat{g}_T f_T^2 \\ -\hat{g}\hat{g}_X f_X^2 & \hat{g}_X^2 f_X^2 & 0 \\ -\hat{g}\hat{g}_T f_T^2 & 0 & \hat{g}_T^2 f_T^2 \end{pmatrix} \]

\[ \Rightarrow T_{3R}^{el} + X^{el} + T_{3R} + X \quad \text{unbroken} \]

\[ B_\mu = \frac{1}{r}(\hat{g}_T \hat{g}_X V_{\mu}^{el} + \hat{g}\hat{g}_X W_{3\mu} + \hat{g}\hat{g}_T X_\mu) \quad r = (\hat{g}_T^2 \hat{g}_X^2 + \hat{g}_X^2 \hat{g}_T^2 + \hat{g}_T^2 \hat{g}_T^2)^{1/2} \]

"composite" \[ \hat{B}_{1\mu}, \hat{B}_{2\mu} \quad \text{of mass} \approx \hat{g}_X^2 f_X^2, \hat{g}_T^2 f_T^2 \]

\[ \mathcal{L}_\Psi = i\bar{f}\gamma^\mu(\partial_\mu - igB_\mu + \ldots) f + i\bar{F}\gamma^\mu(\partial_\mu - igB_\mu + \ldots) F \]

\[ g = \frac{\hat{g}\hat{g}_X \hat{g}_T}{r} \]
Complete model: vectors

\[ SU(3) \times SO(4) \times U(1)_X \]

unbroken:
\[ G^\alpha, W^i_L, B_\mu \]

composite:
\[ \hat{G}^\alpha, \hat{W}^i_L, \hat{W}^\pm_R, \hat{B}_{1\mu}, \hat{B}_{2\mu} \]

\[ SU(4) \times SO(4) \times U(1)_X \]

\[ \rho^A \mu T^A = \left( \begin{array}{ccc} \frac{1}{2} \rho^a \lambda^a & 1 & \frac{1}{\sqrt{2}} V_\mu^i \\ \frac{1}{\sqrt{2}} V_\mu^i & \frac{1}{2 \sqrt{6}} \rho^{15}_\mu & \frac{1}{3 \sqrt{2}} V_\mu \\ \frac{1}{2 \sqrt{6}} \rho^{15}_\mu & -\frac{1}{3 \sqrt{2}} V_\mu \end{array} \right) \]

\[ \sqrt{\frac{2}{3}} T^{15} = \frac{1}{2} (B - L) \]

\[ Y = \sqrt{\frac{2}{3}} T^{15} + T^3_R + X \]

unbroken:
\[ G^\alpha, W^i_L, B_\mu \]

composite as above + \[ V^i_\mu + V^{i+}_\mu + \hat{B}_{3\mu} \]

\[ M_{\hat{G}} \approx M_{\hat{W}_L} \approx M_{\hat{B}_1} \equiv M_{\rho^1} \]
\[ M_{\hat{W}_{R}^\pm} \approx M_{\hat{B}_2} \approx M_{\hat{B}_3} \equiv M_{\rho^2} \]
\[ M_{\rho^1} \approx \frac{\hat{g}_\rho M_W}{g_2 \sqrt{\xi}} \]
\[ \xi = (\frac{v}{f})^2 \]
Complete models: fermions

\[ SU(3) \times SO(4) \times U(1)_X \]

\[ \mathcal{H} = (1, 2, 2)_0 \]
\[ Y = T^3_R + X \]

a possible choice for the composite fermions

\[ Q_U = (3, 2, 2)_{2/3} \quad q_U = (3, 1, 1)_{2/3} \]
\[ Q_D = (3, 2, 2)_{-1/3} \quad q_D = (3, 1, 1)_{-1/3} \]
\[ L = (1, 2, 2)_{-1} \quad l = (1, 1, 1)_{-1} \]

all vector-like. Under the SM gauge group:

\[ Q_U = (3, 2)_{7/6} + (3, 2)_{1/6} \quad q_U = (3, 1, 1)_{2/3} \]
\[ Q_D = (3, 2)_{1/6} + (3, 2)_{-5/6} \quad q_D = (3, 1)_{-1/3} \]
\[ L = (1, 2)_{-1/2} + (1, 2)_{-3/2} \quad l = (1, 1)_{-1} \]
Why not?

\[ SU(3) \times SO(4) \times U(1)_X \]

\[ \mathcal{H} = (1, 2, 2)_0 \]

\[ Y = T^3_R + X \]

\[ Q_L = (3, 2, 1)_{1/6} \]

\[ L_L = (1, 2, 1)_{-1/2} \]

\[ Q_R = (3, 1, 2)_{1/6} \]

\[ L_R = (1, 1, 2)_{-1/2} \]

\[ Q_U = (3, 2, 2)_{2/3} \]

\[ \hat{W}_L Q_L \]

\[ Z \hat{W}_L Q_L \]

\[ + \]

\[ Z \hat{W}_R Q_U \]

\[ = 0 \]
Complete models: fermions

\[ SU(4) \times SO(4) \times U(1)_X \]

\[ \mathcal{H} = (1, 2, 2)_0 \]

\[ Y = \sqrt{\frac{2}{3}} T^{15} + T^3_R + X \]

a possible choice for the composite fermions

\[ \Psi_\pm = (4, 2, 2)_{\pm 1/2} \quad \chi_\pm = (4, 1, 1)_{\pm 1/2} \]

all vector-like. Under the SM gauge group:

\[ \Psi_+ = (3, 2)_{7/6} + (3, 2)_{1/6} + (1, 2)_{1/2} + (1, 2)_{-1/2} \]

\[ \Psi_- = (3, 2)_{1/6} + (3, 2)_{-5/6} + (1, 2)_{-1/2} + (1, 2)_{-3/2} \]

\[ \chi_+ = (3, 1)_{2/3} + (1, 1)_0 \]

\[ \chi_- = (3, 1)_{-1/3} + (1, 1)_{-1} \]
Tree level flavour violations

\[ \mathcal{L}_{eff}^{b \to c\tau\nu} = r_u V_{cb} \left( - \frac{g_2^2}{M_W^2} \right) \left[ \xi s_{Lu3}^2 s_{L\nu3}^2 \right] \left( \frac{1}{2} + \frac{1}{2} f_{W*} \right) (\bar{c}_L \gamma_\mu b_L)(\bar{\tau}_L \gamma_\mu \nu_{3L}) \]

\[ s_{Lu3}^2 s_{L\nu3}^2 = (0.49 \div 0.77) \left( \frac{1.00}{r_u} \right) \left( \frac{0.10}{\xi} \right) \]

\[ \xi = \left( \frac{v}{f} \right)^2 \]

\[ \mathcal{L}_{eff}^{b \to s\mu\mu} = r_d V_{ts} (c_l \epsilon_l)^2 \left( - \frac{g_2^2}{M_W^2} \right) \left[ \xi s_{Lu3}^2 s_{L\nu3}^2 \right] \times \left( 1 + \frac{1}{4} f_{W*} - \frac{1}{12} \frac{3}{2} f_X \right) (\bar{s}_L \gamma_\mu b_L)(\bar{\mu}_L \gamma_\mu \mu_L) \]

\[ s_{Lu3}^2 s_{L\nu3}^2 \approx - (0.65 \div 1.31) \left( \frac{0.07}{c_l^2 \epsilon_l^2} \right) \left( \frac{0.04}{r_d} \right) \left( \frac{0.10}{\xi} \right) \]

\[ \mathcal{L}_{eff}^{\Delta B_s=2} = r_d^2 (V_{ts} V_{tb})^2 \left( - \frac{g_2^2}{M_W^2} \right) \left[ \xi s_{Lu3}^4 \right] \times \left( \frac{1}{2} + \frac{1}{3} f_{G^H} + \frac{1}{4} f_{W*} + \frac{1}{36} \frac{3}{2} f_X \right) (\bar{s}_L \gamma_\mu b_L)^2 \]

\[ r_d^2 s_{Lu3}^4 \left( \frac{\xi}{0.1} \right) \lesssim 2 \cdot 10^{-3} \]
Overall constraints

\[ \Delta B_s = 2 \]

\[ R_D (b \rightarrow cτν) \]

\[ R_K (b \rightarrow sν\bar{ν}) \]

\[ c_l \epsilon_l = 0.25 \]

\[ 0.03 \lesssim \xi s_{Lu3}^2 s_{Lv3}^2 \lesssim 0.09 \]

\[ -0.08 \lesssim r_d \lesssim -0.02 \]

\[ s_{Lu3} = s_{Lv3} \equiv \sin \theta_L \]
LHC Phenomenology

1. Leptoquark pair production

2. Exotic Leptons

3. Resonances in $\tau^+\tau^-$

(Concentrate on 3)
Resonances in $\tau^+ \tau^-$

4 neutral $Z'$-type resonances
the signal crucially depends on $\Gamma_{tot}/M$

\[ SLu3 = SLv3 \]
\[ \xi = 0.1 \]

bands reproduce $R_{D(*)}$ at 1$\sigma$ level
\[ \sigma(pp \rightarrow Z' \rightarrow \tau^+\tau^-) \]

assuming dominance by 1 single \( Z' + \text{SM} \) interference

\[ M_{Z'} \gtrsim 1.1 \div 1.5 \text{ TeV} \]

\[ M_{Z'} \sim g_\rho f / 2 \quad g_\rho \gtrsim 2.8 \div 3.8 \]
The actual \( \sigma(pp \rightarrow \tau^+\tau^-) \)

includes at amplitude level:

- 4 \( Z' \) in the s-channel
- the leptoquark in the t-channel
- the SM contribution

\[
M_{\tau\tau}(\text{GeV})
\]

\[
M_{\rho_1}, M_{\rho_2} = (1.0, 1.5), (1.5, 1.0), (1.0, 1.0), (1.5, 1.5) \text{ TeV}
\]
Conclusion

Let us see if the anomalies get reinforced or fade away
e.g. from the LHCb program
- not only $R_K (B \to Ke^+e^-/B \to K\mu^+\mu^-)$ but similar ratios with different hadronic systems ($K^*$, $\phi$, $\Lambda$, etc.)
- not only $D^*\tau\nu$, but also $D\tau\nu$, $D_s\tau\nu$, $\Lambda_c\tau\nu$, etc.
  - also trying hadronic tau decays

If they are roses ...
  take seriously the leptoquark and $U(2)^5$
  and perhaps a composite picture
An “Extreme Flavour” experiment?

- Currently planned experiments at the HL-LHC will only exploit a small fraction of the huge rate of heavy-flavoured hadrons produced
  - ATLAS/CMS: full LHC integrated luminosity of 3000 fb\(^{-1}\), but limited efficiency due to lepton high \(p_T\) requirements
  - LHCb: high efficiency, also on charm events and hadronic final states, but limited in luminosity, 50 fb\(^{-1}\) vs 3000 fb\(^{-1}\)

- Would an experiment capable of exploiting the full HL-LHC luminosity for flavour physics be conceivable?
  - Aiming at collecting \(O(100)\) times the LHCb upgrade luminosity
    \(\Rightarrow 10^{14}\) b and \(10^{15}\) c hadrons in acceptance at \(L=10^{35}\) cm\(^{-2}\)s\(^{-1}\)

**Motivation:** test CKM (FCNC loops) from \(\approx 20\%\) to \(\approx 1\%\)
A minimal list of key observables in QFV to be improved and not yet TH-error dominated

- $\gamma$ from tree: $B \to D K$, etc (now better from loops)
- $|V_{ub}|, |V_{cb}|$
- $B \to \tau\nu, \mu\nu \ (+D^{(*)})$
- $B \to K^{(*)} l^+ l^-, \nu\nu$ (in suitable observables?)
- $K_S, D, B_{s,d} \to l^+ l^-$ ("Higgs penguins")
- $\phi^\Delta_{d,s}$ (CPV in $\Delta B_{d,s} = 2$)
- $K^+, K_L \to \pi\nu\nu$
- $\Delta A_{CP}$ in selected $D$ modes