Supergravity UV properties and Color-Kinematics Duality

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Supersymmetry, Quantum Gravity and Gauge Fields

Pisa

1106.4711, 1201.5366, 1207.6666
(and ongoing work)

Collaborators: Zvi Bern, Lance Dixon, John Joseph Carrasco, Radu Roiban
Motivation

- ~35 years after the birth of supergravity, the detailed knowledge of potential $D=4$ UV divergences are within reach.
- No $D=4$ divergence of pure SG has been found to date.
- Susy forbids 1,2 loop div., $R^2$, $R^3$ c.t. incompatible with susy
- Pure gravity 1-loop finite, 2-loop divergent Goroff & Sagnotti
- $\mathcal{N}=8$ SG best candidate for a UV finite QFT
- No divergence before 7 loops
- 5-loop divergence in $D=24/5$
  predicts 7-loop divergence in $D=4$.

- We don’t have result yet...but soon!
Motivation

- If $\mathcal{N}=8$ SG is perturbatively finite, why is it interesting?
- It better be finite for a good reason!
  - Hidden new symmetry, for example
  - Understanding the mechanism might open a host of possibilities

- Any indication of hidden structures yet?
  - Gravity is a double copy of gauge theories
  - The gauge theories behave as Lie algebras
  - Color-Kinematics Duality  Bern, Carrasco, HJ

Gravity

Symmetry?
Outline

- Review UV status N=8 SG
- New 5-loop SYM amplitude, stepping stone to N=8.
- Duality between color and kinematics
- Double-copy structure of gravity
- Loop amplitudes with manifest duality
  - UV properties at 4 and 5 pts
  - R-symmetry violating amplitudes
- Conclusion
UV properties $\mathcal{N}=8$ SG

In D=4 dimensions:

• $\mathcal{N}=8$ SG: conventional superspace power counting forbids $L=1,2$ divergences Green, Schwarz, Brink, Howe and Stelle, Marcus and Sagnotti


• $L<7$ loop divergences ruled out by counterterm analysis, using $E_7(7)$ symmetry and other methods, but a $L=7$ divergence is still possible Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Björnsson, Green, Bossard, Howe, Stelle, Vanhove, Kallosh, Ramond, Lindström, Berkovits, Grisaru, Siegel, Russo, and more....

In D>4 dimensions:

Through four loops $\mathcal{N}=8$ SG and $\mathcal{N}=4$ SYM diverge in exactly the same dimension:

$$D_c = 4 + \frac{6}{L} \quad (L > 1)$$

Marcus and Sagnotti; Bern, Dixon, Dunbar, Perelstein, Rozowsky; Bern, Carrasco, Dixon, HJ, Kosower, Roiban
UV divergence trend

Plot of critical dimensions of $\mathcal{N} = 8$ SUGRA and $\mathcal{N} = 4$ SYM

1-2 loops: Green, Schwarz, Brink; Marcus and Sagnotti
3-5 loops: Bern, Carrasco, Dixon, HJ, Kosower, Roiban

$L = 7$ lowest loop order for possible $D = 4$ divergence

Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger;
Björnsson, Green, Bossard, Howe, Stelle, Vanhove Kallosh, Ramond, Lindström, Berkovits, Grisaru, Siegel, Russo, and more....
<table>
<thead>
<tr>
<th>Loop order</th>
<th>4pt amplitude form (any dimension)</th>
<th>divergence occurs in</th>
<th>Counter term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$R^4 \times \text{(intgrl.)}$</td>
<td>$D = 8$</td>
<td>$\sim R^4$</td>
</tr>
<tr>
<td>2</td>
<td>$\partial^4 R^4 \times \text{(intgrl.)}$</td>
<td>$D = 7$</td>
<td>$\sim \partial^4 R^4$</td>
</tr>
<tr>
<td>3</td>
<td>$\partial^6 R^4 \times \text{(intgrl.)}$</td>
<td>$D = 6$</td>
<td>$\sim \partial^6 R^4$</td>
</tr>
<tr>
<td>4</td>
<td>$\partial^8 R^4 \times \text{(intgrl.)}$</td>
<td>$D = 5.5$</td>
<td>$\sim \partial^8 R^4$</td>
</tr>
<tr>
<td>5</td>
<td>$\partial^{10} R^4 \times \text{(intgrl.)}$</td>
<td>$D = 26/5$ ?</td>
<td>$\sim \partial^{10} R^4$ ?</td>
</tr>
</tbody>
</table>

If amplitude for $L \geq 5$ has at least 10 derivatives then by dimensional analysis:
no divergence before $L = 8$ !

$M_{stu}^\text{tree} \sim R^4$

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## Opinions on First $\mathcal{N} = 8$ Divergence

<table>
<thead>
<tr>
<th>Loops</th>
<th>Opinions</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 loops</td>
<td>If $\mathcal{N} = 7$ harmonic superspace exists</td>
</tr>
<tr>
<td>7 loops</td>
<td>If $\mathcal{N} = 8$ harmonic superspace exists; lightcone gauge locality arguments; Hints from string U-duality analysis. $E_{7(7)}$ analysis.</td>
</tr>
<tr>
<td>8 loops</td>
<td>Explicit identification of potential susy invariant counterterm with full non-linear susy</td>
</tr>
<tr>
<td>9 loops</td>
<td>Assume Berkovits’ superstring non-renormalization theorems can be carried over to $\mathcal{N} = 8$ supergravity</td>
</tr>
<tr>
<td>Finite</td>
<td>Identified cancellations in multiloop amplitudes; lightcone gauge locality and $E_{7(7)}$</td>
</tr>
</tbody>
</table>

**Note:** no divergence demonstrated above. Arguments based on lack of susy protection!
\[ N = 4 \] SYM 5-loop Amplitude

\[ N = 4 \] SYM important stepping stone to \[ N = 8 \] SG

1207.6666 [hep-th]
Bern, Carrasco, HJ, Roiban

• 416 integral topologies:

• Used maximal cut method
  Bern, Carrasco, HJ, Kosower

• Maximal cuts: 410
• Next-to-MC: 2473
• \( N^2 \)MC: 7917
• \( N^3 \)MC: 15156

Unitarity cuts done in \( D \) dimensions...integrated UV div. in \( D = 26/5 \)

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\[ N = 4 \text{ SYM 5-loop UV divergence} \]

Non-Planar UV divergence in \( D=26/5 \):

\[
A_{4}^{(5)} \bigg|_{\text{div}} = - \frac{144}{5} g^{12} st A_{4}^{\text{tree}} N_{c}^{3} \left( N_{c}^{2} V^{(a)} + 12(V^{(a)} + 2V^{(b)} + V^{(c)}) \right) \\
\times \text{Tr}[T^{a_{1}} T^{a_{2}} T^{a_{3}} T^{a_{4}}]
\]

Double traces and single-trace NNLC finite in \( D=26/5 \), only single-trace LC and NLC are divergent

\[
N_{c}^{1} \text{ Tr}[T^{a_{1}} T^{a_{2}} T^{a_{3}} T^{a_{4}}] \\
N_{c}^{4} \text{ Tr}[T^{a_{1}} T^{a_{2}}] \text{ Tr}[T^{a_{3}} T^{a_{4}}] \\
N_{c}^{2} \text{ Tr}[T^{a_{1}} T^{a_{2}}] \text{ Tr}[T^{a_{3}} T^{a_{4}}] \\
N_{c}^{0} \text{ Tr}[T^{a_{1}} T^{a_{2}}] \text{ Tr}[T^{a_{3}} T^{a_{4}}]
\]

Bern, Carrasco, HJ, Roiban

Vanish!

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Color-Kinematics Duality
Yang-Mills theories are controlled by a kinematic Lie algebra

- Amplitude represented by cubic graphs:

\[ \mathcal{A}_m^{(L)} = \sum_{i \in \Gamma_3} \int \frac{d^{LD} \ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i1}^2 p_{i2}^2 p_{i3}^2 \ldots p_{iL}^2} \]

Color & kinematic numerators satisfy same relations:

- Jacobi identity
- Antisymmetry

Duality: color ↔ kinematics

Bern, Carrasco, HJ

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Some details of color-kinematics duality

can be checked for 4pt on-shell ampl. using Feynman rules

Example with two quarks:

\[
\begin{align*}
\varepsilon_2 \cdot (\bar{u}_1 V u_3) \cdot \varepsilon_4 &= \bar{u}_1 f^p q u_3 - \bar{u}_1 f^p q u_3
\end{align*}
\]

1. \((A^\mu)^4\) contact interactions absorbed into cubic graphs
   - by hand \(1=s/s\)
   - or by auxiliary field \(B \sim (A^\mu)^2\)

2. Beyond 4-pts duality not automatic \(\rightarrow\) Lagrangian reorganization

3. Known to work at tree level: all-\(n\) example Kiermaier; Bjerrum-Bohr et al.

4. Enforces (BCJ) relations on partial amplitudes \(\rightarrow\) \((n-3)!\) basis

5. Same/similar relations control string theory S-matrix

Bjerrum-Bohr, Damgaard, Vanhove; Stieberger
Gravity is a double copy

- Gravity amplitudes obtained by replacing color with kinematics

\[ A^{(L)}_m = \sum_{i \in \Gamma_3} \int \frac{d^{LD} \ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2} \]

\[ M^{(L)}_m = \sum_{i \in \Gamma_3} \int \frac{d^{LD} \ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2} \]

- The two numerators can belong to different theories:

<table>
<thead>
<tr>
<th>Theory 1</th>
<th>Theory 2</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N=4)</td>
<td>(N=4)</td>
<td>(N=8) sugra</td>
</tr>
<tr>
<td>(N=4)</td>
<td>(N=2)</td>
<td>(N=6) sugra</td>
</tr>
<tr>
<td>(N=4)</td>
<td>(N=0)</td>
<td>(N=4) sugra</td>
</tr>
<tr>
<td>(N=0)</td>
<td>(N=0)</td>
<td>Einstein gravity + axion + dilaton</td>
</tr>
</tbody>
</table>

Similar to Kawai-Lewellen-Tye but works at loop level
BLG color-kinematics

*D=3* Chern-Simons matter theories obey color-kinematics duality.

Gauge group is a 3-algebra:

\[
[T^a, T^b, T^c] = f^{abc}_d T^d
\]

Fundamental identity (Jacobi identity):

\[
C_s = C_t + C_u + C_v \iff n_s = n_t + n_u + n_v
\]

4 and 6 point checks shows that the double copy of BLG is *E*$_8$(8) *N* = 16 SG of Marcus and Schwarz

BLG =‘square root’ of N=16 SG

\[
A_4^{\text{BLG}} = \sqrt{M_4^{N=16}} = \sqrt{\frac{\delta^{16}(Q)}{stu}}
\]

Bargheer, Lamberti, and Gustavsson

Bargheer, He, and McLoughlin

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Self-Dual Kinematic Algebra

Self dual YM in light-cone gauge:

Generators of diffeomorphism invariance:

\[ L_k = e^{-ik \cdot x} (-k_w \partial_u + k_u \partial_w) \]

Lie Algebra:

\[ [L_{p_1}, L_{p_2}] = i X(p_1, p_2) L_{p_1 + p_2} = i F_{p_1 p_2}^k L_k \]

The \( X(p_1, p_2) \) are YM vertices of type ++- helicity.

Diffeomorphism symmetry hidden in YM theory!

Self dual sector gives +++...+ amplitudes: only one-loop S-matrix.
We need to find the algebra beyond that.
Loop amplitudes with CK duality
Manifest C-K amplitudes

- $\mathcal{N}=4$ SYM and $\mathcal{N}=8$ SG
  - Tree level
  - MHV sector at loop level
  - R-symmetry violating ampls

Tree level:
- All-$n$ representation of numerators via KLT
  Kiermaier, Bjerrum-Bohr, Damgaard, Sondergaard Vanhove
- Duality-satisfying Feynman rules for MHV
  Monteiro and O'Connell

- Pure QCD and Einstein gravity
  - Tree level
  - All-plus helicity ampls
  - Dilaton, axion, in loops
C-K amplitudes, 1-loop examples

Known duality-satisfying loop amplitudes:

N=4 SYM:

\[ \beta \]

All-plus QCD:

\[ \gamma \]

N=4 SYM and All-plus QCD:

\[ \beta_{12345}^{N=4} = \delta^{(8)}(Q) \frac{[12][23][34][45][51]}{4\epsilon(1, 2, 3, 4)} \]

\[ \gamma_{12}^{N=4} = \delta^{(8)}(Q) \frac{[12]^2[34][45][35]}{4\epsilon(1, 2, 3, 4)} \]

\[ \beta_{12345}^{+++} = \mu^4 \frac{[12][23][34][45][51]}{4\epsilon(1, 2, 3, 4)} \]

\[ \gamma_{12}^{+++} = \mu^4 \frac{[12]^2[34][45][35]}{4\epsilon(1, 2, 3, 4)} \]
C-K amplitudes, 2-loop examples

\[ \mathcal{A}_4^{\text{tree}} \left( \begin{array}{ccc}
2 & 3 \\
\text{s} & 1 & 4 \\
\end{array} \right) + \mathcal{A}_4^{\text{tree}} \left( \begin{array}{ccc}
1 & 2 & 3 \\
\text{s} & & 4 \\
\end{array} \right) + \text{perms} \]

\text{N=4 SYM} \hspace{1cm} \text{Bern, Dixon, Kosower (2000); Bern, Carrasco, HJ}

\text{All-plus QCD} \hspace{1cm} \text{Bern, Dixon, Dunbar, Perelstein and Rozowsky (1998)}

\[
\frac{[1 \ 2] [3 \ 4]}{\langle 1 \ 2 \rangle \langle 3 \ 4 \rangle} \ s_{12} \mathcal{I}_4^p \left( (D_s - 2)(\lambda_p^2 \lambda_q^2 + \lambda_p^2 \lambda_{p+q}^2 + \lambda_q^2 \lambda_{p+q}^2) + 16(\lambda_p \cdot \lambda_q)^2 - \lambda_p^2 \lambda_q^2 \right)
\]

\text{Squaring gives all-plus-helicity Einstein gravity amplitude (with dilation and axion in loops)}

Trento 17/07/12 H. Johansson
3-loop N=4 SYM

manifest realization of duality

$\mathcal{N}=8$ SG is simply the square

<table>
<thead>
<tr>
<th>Integral $I^{(\sigma)}$</th>
<th>$\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}}=8$ supergravity) numerator</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)−(d)</td>
<td>$s^2$</td>
</tr>
<tr>
<td>(e)−(g)</td>
<td>$(s(-\tau_{35} + \tau_{45} + t) - t(\tau_{25} + \tau_{45})) + u(\tau_{25} + \tau_{35}) - s^2)/3$</td>
</tr>
<tr>
<td>(h)</td>
<td>$(s(2\tau_{15} - \tau_{16} + 2\tau_{26} - \tau_{27} + 2\tau_{35} + \tau_{36} + \tau_{37} - u) + t(\tau_{16} + \tau_{26} - \tau_{37} + 2\tau_{36} - 2\tau_{15} - 2\tau_{27} - 2\tau_{35} - 3\tau_{17}) + s^2)/3$</td>
</tr>
<tr>
<td>(i)</td>
<td>$(s(-\tau_{25} - \tau_{26} - \tau_{35} + \tau_{36} + \tau_{45} + 2t) + t(\tau_{26} + \tau_{35} + 2\tau_{36} + 2\tau_{45} + 3\tau_{46}) + u\tau_{25} + s^2)/3$</td>
</tr>
<tr>
<td>(j)−(l)</td>
<td>$s(t-u)/3$</td>
</tr>
</tbody>
</table>

$\tau_{ij} = 2k_i \cdot l_j$

Used to show absence of $\mathcal{N}=4$ SG divergence  

Bern, Davies, Dennen, Huang
2-loop 5-pts $\mathcal{N}=4$ SYM and $\mathcal{N}=8$ SG

The 2-loop 5-point amplitude with duality exposed

$\mathcal{N}=8$ SG obtained from numerator double copies

<table>
<thead>
<tr>
<th>$I^{(x)}$</th>
<th>$\mathcal{N} = 4$ Super-Yang-Mills ($\sqrt{\mathcal{N}} = 8$ supergravity) numerator</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a),(b)</td>
<td>$\frac{1}{4} \left( \gamma_{12} (2 s_{45} - s_{12} + \tau_{2p} - \tau_{1p}) + \gamma_{23} (s_{45} + 2 s_{12} - \tau_{2p} + \tau_{3p}) + 2 \gamma_{45} (\tau_{5p} - \tau_{4p}) + \gamma_{13} (s_{12} + s_{45} - \tau_{1p} + \tau_{3p}) \right)$</td>
</tr>
<tr>
<td>(c)</td>
<td>$\frac{1}{4} \left( \gamma_{15} (\tau_{5p} - \tau_{1p}) + \gamma_{25} (s_{12} - \tau_{2p} + \tau_{5p}) + \gamma_{12} (s_{34} + \tau_{2p} - \tau_{1p} + 2 s_{15} + 2 \tau_{1q} - 2 \tau_{2q}) + \gamma_{45} (\tau_{4q} - \tau_{5q}) - \gamma_{35} (s_{34} - \tau_{3q} + \tau_{5q}) + \gamma_{34} (s_{12} + \tau_{3q} - \tau_{4q} + 2 s_{45} + 2 \tau_{4p} - 2 \tau_{3p}) \right)$</td>
</tr>
<tr>
<td>(d)-(f)</td>
<td>$\gamma_{12} s_{45} - \frac{1}{4} \left( 2 \gamma_{12} + \gamma_{13} - \gamma_{23} \right) s_{12}$</td>
</tr>
</tbody>
</table>

$\tau_{ip} = 2k_i \cdot p$

Carrasco, HJ
1106.4711 [hep-th]

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1-loop 5-pts UV divergences

\[ \beta_{12345} \equiv N^{(P)} = \delta^{(8)}(Q) \frac{[12][23][34][45][51]}{4 \varepsilon(1,2,3,4)} \]

\[ \gamma_{12} \equiv N^{(B)} = \delta^{(8)}(Q) \frac{[12]^2[34][45][35]}{4 \varepsilon(1,2,3,4)} \]

**SYM UV div in D=8:**

\[ \mathcal{A}^{(1)}_5 \bigg|_{UV} = -g^5 \frac{1}{6(4\pi)^4 \varepsilon} \left[ N_c \text{ Tr}_{12345} \left( \frac{\gamma_{12}}{s_{12}} + \frac{\gamma_{23}}{s_{23}} + \frac{\gamma_{34}}{s_{34}} + \frac{\gamma_{45}}{s_{45}} + \frac{\gamma_{51}}{s_{51}} \right) 
+ 6 \text{ Tr}_{123} \text{ Tr}_{45} \left( \frac{\gamma_{12}}{s_{12}} + \frac{\gamma_{23}}{s_{23}} + \frac{\gamma_{31}}{s_{31}} \right) + \text{ perms} \right] \]

**SG UV div in D=8:**

\[ \mathcal{M}^{(1)}_5 \bigg|_{UV} = -\frac{\kappa}{2} \frac{1}{6(4\pi)^4 \varepsilon} \left[ \frac{\gamma_{12}^2}{s_{12}} + \frac{\gamma_{13}^2}{s_{13}} + \frac{\gamma_{14}^2}{s_{14}} + \frac{\gamma_{15}^2}{s_{15}} + \frac{\gamma_{23}^2}{s_{23}} + \frac{\gamma_{24}^2}{s_{24}} + \frac{\gamma_{25}^2}{s_{25}} + \frac{\gamma_{34}^2}{s_{34}} + \frac{\gamma_{35}^2}{s_{35}} + \frac{\gamma_{45}^2}{s_{45}} \right] \]

**SU(8) violating SG UV div in D=8:**

\[ \mathcal{M}^{(1)}_5 \bigg|_{UV} = \frac{\kappa}{2} \frac{1}{3(4\pi)^4 \varepsilon} \left( \delta^{(4)}(Q) \delta^{(4)}(\bar{Q}) + \delta^{(4)}(\bar{Q}) \delta^{(4)}(\bar{Q}) \right) \]

**counterterms:** \( \text{Tr}[F^4], (\text{Tr}[F^2])^2, R^4, \phi R^4 \)
2-loop 5-pts UV divergences

SYM UV div in $D=7$:

$$\mathcal{A}_5^{(2)}\big|_{\text{UV}} = -g^7 \left[ (N_c^2 V^{(P)} + 12(V^{(P)} + V^{(NP)})) \text{Tr}_{12345} \left( 5\beta_{12345} + \frac{\gamma_{12}}{s_{12}} (s_{35} - 2s_{12}) 
+ \frac{\gamma_{23}}{s_{23}} (s_{14} - 2s_{23}) + \frac{\gamma_{34}}{s_{34}} (s_{25} - 2s_{34}) 
+ \frac{\gamma_{45}}{s_{45}} (s_{13} - 2s_{45}) + \frac{\gamma_{51}}{s_{15}} (s_{24} - 2s_{15}) \right) 
- 12N_c(V^{(P)} + V^{(NP)}) \text{Tr}_{123} \text{Tr}_{45} s_{45} \left( \frac{\gamma_{12}}{s_{12}} + \frac{\gamma_{23}}{s_{23}} + \frac{\gamma_{31}}{s_{31}} \right) + \text{perms} \right]$$

SG UV div in $D=7$:

$$\mathcal{M}_5^{(2)}\big|_{\text{UV}} = i \left( \frac{\kappa}{2} \right)^7 \frac{1}{6} (V^{(P)} + V^{(NP)}) \sum_{S_5} \frac{\gamma_{12}}{s_{12}} \left( s_{34}^2 + s_{35}^2 + s_{45}^2 - 3s_{12}^2 \right) \sim D^4 R^4$$

SU(8) violating SG UV div in $D=8$:

$$\mathcal{M}_5^{(2)}\big|_{\text{UV}} = \frac{3i}{4} \left( \frac{\kappa}{2} \right)^7 \sum (V^{(P)} + V^{(NP)}) \left( s_{12}^2 + s_{13}^2 + s_{14}^2 + s_{15}^2 + s_{23}^2 + s_{24}^2 + s_{25}^2 + s_{34}^2 + s_{35}^2 + s_{45}^2 \right) \sim D^4 \phi R^4$$

Carrasco, HJ \textbf{1106.4711} [hep-th]
4-loops $\mathcal{N}=4$ SYM and $\mathcal{N}=8$ SG

$N^\text{SYM}_6 = \frac{1}{2} s_{12}^2 (\tau_{45} - \tau_{35} - s_{12})$

$N^\text{SYM}_8 = \left[ \frac{1}{2} s_{12}^2 (\tau_{45} - \tau_{35} - s_{12}) \right]^2$

$N^\text{SG}_6 = \left[ \frac{1}{2} s_{12}^2 (s_{13} - s_{23}) \right]^2$

$N^\text{SG}_8 = \left[ 16 s_{12}^2 (s_{13} - s_{23}) \right]^2$

- 85 diagrams with duality manifest
- Power counting manifest both $\mathcal{N}=4$ and $\mathcal{N}=8$
- Both diverge in $D=11/2$

$A_4^{(4)}(1, 2, 3, 4)_{\text{pole}}^{SU(N_c)} = -6 g^{10} \mathcal{K} N_c^2 (N_c^2 V_1 + 12 (V_1 + 2 V_2 + V_8))$

$\times \left( s (\text{Tr}_{1324} + \text{Tr}_{1423}) + t (\text{Tr}_{1243} + \text{Tr}_{1342}) + u (\text{Tr}_{1234} + \text{Tr}_{1432}) \right)$

$M_4^{(4)}_{\text{pole}} = -\frac{23}{8} \left( \frac{\kappa}{2} \right)^{10} s_{12} s_{13} s_{23} (s_{12}^2 + s_{13}^2 + s_{23}^2)^2 M_4^{\text{tree}} (V_1 + 2 V_2 + V_8)$

up to overall factor, divergence same as for $\mathcal{N}=4$ $1/N_c^2$ part

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Summary and Outlook

- Explicit calculations in $\mathcal{N}=8$ SUGRA up to four loops show that the power counting exactly follows that of $\mathcal{N}=4$ SYM – a finite theory.

- $E_7$ arguments, supersymmetry and string theory insight have shown that a divergence appears no earlier than 7 loops.

- 5 loop calculation in $D=24/5$ probes the same counterterm as the 7-loop $D=4$ amplitude. It will provide critical input to the N=8 question.

- Color-Kinematics Duality allows us to do gravity calculations simply by reorganizing the Yang-Mills amplitude (or even BLG amplitude).

- A faithful representation of the kinematic algebra is still missing for all but the simplest case of self-dual Yang-Mills.

- Constructing CK-amplitude representations is nonetheless possible, case by case. The double-copy formula then gives gravity integrands for free, greatly facilitating UV analysis.

- Stay tuned for the 5-loop result...