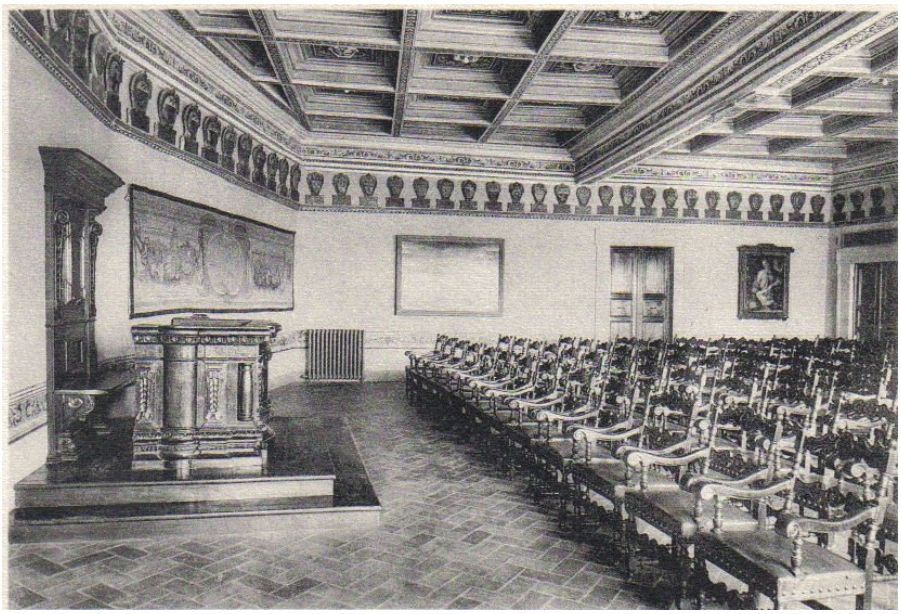
Variational Methods in Analysis, Geometry and Physics

February 12–16, 2018



Scuola Normale Superiore, Sala degli Stemmi

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Abstracts

Nonlinear Schrödinger Equation on Quantum Graph: existence of ground states

Riccardo Adami
Politecnico di Torino

We review on some recent results concerning the existence of ground states for the Schrödinger equation with subcritical and critical nonlinearity power, defined either on quantum graphs made of a finite number of vertices and at least one infinte edge, or on periodic graphs. This is a joint project with Simone Dovetta, Enrico Serra, and Paolo Tilli.

Fractional periodic problems: existence and multiplicity

Vincenzo Ambrosio
Università degli studi di Urbino

In this talk we present some recent existence and multiplicity results for different fractional T-periodic problems. Such equations are driven by the nonlocal operator $(\Delta+m^s)^s$, where $s \in (0,1)$ and $m \geq 0$, defined through the spectral decomposition of the elliptic operator $\Delta + m^2$ with periodic boundary conditions. We investigate these problems using suitable variational methods after transforming them into degenerate elliptic equations in the half-cylinder $(0,T)^N \times (0,\infty)$, with periodic conditions on the lateral boundary $\partial(0,T)^N \times [0,\infty)$ and a nonlinear Neumann boundary condition on $(0,T)^N \times \{0\}$.

Entire solutions for Liouville systems

Luca Battaglia
"Sapienza" Università di Roma

I will consider a system of two coupled Liouville equations on the plane. The system admits so-called scalar solutions, namely such that the two components coincide. These solutions actually solve a scalar Liouville equation on the plane, hence they are very well known and they have been completely classified. On the other hand, much less is known about non-scalar solutions. Using bifurcation theory, I will show the existence of some branches of (non-scalar) solutions bifurcating from a scalar solution. This is a joint work with Francesca Gladiali and Massimo Grossi.

Real analytic hypoellipticity for sums of squares: an assessment

Antonio Bove
Università di Bologna

We consider sums of squares of vector fields with real analytic coefficients satisfying Hörmander's condition. We exhibit a counterexample to the Treves conjecture and discuss a possible direction leading to the definition of a stratification of the characteristic variety. Analytic hypoellipticity should occur when the strata of the stratification are real analytic symplectic submanifolds. The case of a class of operators having a "single" symplectic stratum is discussed in detail.

The Poisson equation on Riemannian manifolds

Giovanni Catino
Politecnico di Milano

In this talk I will present some recent results concerning the existence of solutions to the classical Poisson equation on Riemannian manifolds. This is a joint work with D.D. Monticelli and F. Punzo.

Schauder estimates at the boundary in Carnot groups

Giovanna Citti
Università di Bologna

I'll present a new approach to prove Schauder estimates at the boundary for sub-Laplacian type operators in Carnot groups. While internal Schauder estimates have been deeply studied, up to now subriemannian estimates at the boundary are known only in the Heisenberg groups, but the technique used in this case cannot be extended to general Carnot groups. I'll present a new method, obtained in collaboration with Baldi and Cupini which allows to build a Poisson kernel starting from the fundamental solution, from which we deduce the Schauder estimates at non-characteristic boundary points.

Exponential PDE's in high dimension

Pierpaolo Esposito
Università di Roma Tre

In dimension $n \geq 2$ elliptic PDE's with an exponential non-linearity can have a critical behavior when the differential operator either involves derivatives of order > 2 or is quasi-linear. Due to intrinsic invariances such PDE's present a natural lack of compactness and in the quasi-linear case I aim to present some recent results concerning existence issues and the description of the blow-up mechanism. In the last part of the talk, I will report on an ongoing research project, in collaboration with A. Malchiodi, concerning a four-dimensional PDE arising in the theory of log-determinants in conformal geometry, where the differential operator is fourth-order and quasi-linear at the same time.

A sharp Bernstein-type theorem for entire minimal graphs

Alberto Farina
Université de Picardie Jules Verne

We consider entire solutions u to the minimal surface equation in $\mathbb{R}^N$, with $N \geq 8$, and we prove the following sharp result: *if $N-7$ partial derivatives $\frac{\partial u}{\partial x_j}$ are bounded on one side (not necessarily the same), then u is necessarily an affine function.*

Local well-posedness for quasi-linear NLS with large Cauchy data on the circle

Roberto Feola
SISSA

I discuss local in time well-posedness for a large class of quasi-linear Hamiltonian, or parity preserving, Schrödinger equation on the circle. Using para-differential tools one can show that the system can be reduced to another one with symbols which, at the positive order, are constant and purely imaginary. This allows to obtain a priori energy estimates on the Sobolev norms of the solution.

Quasilinear Elliptic problems in Perugia

Roberta Filippucci
Università di Perugia

I will briefly present some of the results obtained by the team of the PRIN project in Perugia which concern elliptic problems of variational and non variational type. In particular, we illustrate both existence and nonexistence theorems, settled in Euclidean spaces, in Heisenberg and Carnot groups.

Poincaré inequalities for differential forms on Heisenberg group

Bruno Franchi
Università di Bologna

Every closed differential form ω on a Euclidean ball has a primitive whose L^q norm is bounded by the L^p norm of ω (for suitable exponents p and q). We prove an analogous result for Rumin's exterior differential on Heisenberg balls. This is used to prove vanishing of $\ell^{q,p}$-cohomology of Heisenberg groups. Joint work with Annalisa Baldi and Pierre Pansu.

Nonlinear beams modeling suspension bridges: spectral theory and instability

Maurizio Garrione
Politecnico di Milano

We consider the fourth-order equation

$$u_{tt} + u_{xxxx} + f(u) = g(x,t), \tag{1}$$

complemented with Navier boundary conditions, modeling a beam subject to a nonlinear restoring force f and an external force g. We recall the "application-oriented" notion of instability for (1) defined in [M. Garrione and F. Gazzola, 2017], inspired also by the Tacoma Narrows Bridge collapse in 1940; it embodies a sudden and significant loss of energy concentration from a prevailing mode of oscillation to a residual one. We compare the analysis of this kind of instability in a free beam with the one in a beam with intermediate piers (for which some considerations about the linear problem are in order, as well), discussing some numerical experiments.

Quasi-radial solutions for the Lane-Emden problem in the ball

Isabella Ianni
Università degli Studi della Campania Luigi Vanvitelli

We consider the semilinear elliptic problem

$$\begin{cases} -\Delta u = |u|^{p-1}u & \text{in } B \\ u = 0 & \text{on } \partial B \end{cases} \tag{$\mathscr{E}_p$}$$

where B is the unit ball of $\mathbb{R}^2$ centered at the origin and $p \in (1, +\infty)$. We show the existence of nonradial sign-changing solutions to $(\mathscr{E}_p)$ which are *quasi-radial*, namely solutions whose nodal line is the union of a finite number of disjoint simple closed curves, which are the boundary of nested domains contained in B. In particular their nodal line doesn't touch ∂B. The result is obtained both via nonradial bifurcation from the least energy sign-changing radial solution of $(\mathscr{E}_p)$ at certain values of p and by considering, for p large, the least energy nodal solutions in spaces of functions invariant by the action of the dihedral group generated by the reflection with respect to the x-axis and the rotation about the origin of angle $\frac{2\pi}{k}$, for suitable integers k.

We also show that for certain integers k the least energy nodal solutions in these spaces of symmetric functions are instead radial, exhibiting a breaking of symmetry phenomenon in dependence on the exponent p. The results are obtained with F. Gladiali (University of Sassari, Italy).

Uniqueness of solutions to singular Liouville-type equations

Alex Jevnikar
Università di Pisa

We deduce new uniqueness results for solutions to singular Liouville-type equations both on spheres and on bounded domains, as well as new self-contained proofs of previously known results. To this end, we derive a singular Sphere Covering Inequality based on the singular Alexandrov-Bol isoperimetric inequality and symmetric rearrangements. Work in collaboration with D. Bartolucci, C. Gui and A. Moradifam.

On some Liouville-Bernstein theorems for graphs with prescribed mean curvature

Luciano Mari
Scuola Normale Superiore

The core of this talk is the investigation of Bernstein-Liouville type theorems for entire graphs $u\colon M \to \mathbb{R}$ over a complete Riemannian manifold, under the condition that the mean curvature of the graph is a prescribed function. Emphasis will be put on minimal graphs, on solitons for the mean curvature flow and on capillary graphs. After a review of some classical results to stress the peculiar features of the mean curvature operator, we will focus on recent contributions based on joint works with B. Bianchini, M. Rigoli and P. Pucci. The geometry of the underlying manifold is taken into account just via the growth of the volume of its geodesic balls. The main Liouville type theorems are also applicable to more general quasilinear inequalities of the type

$$\operatorname{div}\big(A(|\nabla u|)\nabla u\big) \geq b(x)f(u)l(|\nabla u|),$$

a model that includes operators of interest like the one of exponentially harmonic functions, and the p and (p,q)-Laplacians.

Long time stability of small finite gap solutions of the cubic Nonlinear Schrödinger equation on $\mathbb{T}^2$

Alberto Maspero
SISSA

We study long time stability of a class of nontrivial, quasi-periodic solutions depending on one spacial variable of the cubic defocusing non-linear Schrödinger equation on the two dimensional torus. We prove that these quasi-periodic solutions are orbitally stable for finite but long times, provided that their Fourier support and their frequency vector satisfy some complicated but explicit condition, which we show holds true for most solutions. This is a joint work with M. Procesi.

Nonlinear problems with lack of compactness

Giovanni Molica Bisci
Università Mediterranea di Reggio Calabria

A very interesting area of nonlinear analysis lies in the study of equations involving critical (in the sense of Sobolev embedding) nonlinearities. A great attention has been focused on these problems, both for the pure mathematical research and in view of concrete real–world applications. In particular, critical equations are relevant for their relations with problems arising in differential geometry and in physics, where a lack of compactness occurs (see, for instance, [FMS, MaMo, MRS, MoSe]). In this talk, motivated by this wide interest in the current literature, also in connection with the celebrated Brezis–Nirenberg problem and inspired by the recent papers [BoPu] and [Pu], we present some existence results on unbounded domains of the Heisenberg group (see [MoPu0] and [MoPu1]). Certain open problems will be briefly presented.

Bibliography

[BoPu] S. BORDONI AND P. PUCCI, *Schrödinger–Hardy systems involving two Laplacian operators in the Heisenberg group*, submitted for publication, pages 29.

[FMS] A. FISCELLA, G. MOLICA BISCI, AND R. SERVADEI, *Bifurcation and multiplicity results for critical nonlocal fractional Laplacian problems*, Bull. Sci. Math. **140** (2016), 14-35.

[MaMo] J. MAWHIN AND G. MOLICA BISCI, *A Brezis–Nirenberg type result for a nonlocal fractional operator*, J. Lond. Math. Soc. **95** (2017), 73-93.

[MoPu0] G. MOLICA BISCI AND P. PUCCI, *Critical equations on $\mathscr{H}$ domains of Carnot groups*, submitted for publication, pages 18.

[MoPu1] G. MOLICA BISCI AND P. PUCCI, *Subelliptic critical equations on unbounded domains of the Heisenberg group*, in preparation.

[MRS] G. MOLICA BISCI, V. RĂDULESCU, AND R. SERVADEI, Variational Methods for Nonlocal Fractional Problems, Encyclopedia of Mathematics and its Applications, No. **162**, *Cambridge University Press*, Cambridge, 2016.

[MoSe] G. MOLICA BISCI AND D. REPOVS, *A Yamabe–type problem on Carnot groups*, Potential Anal. **46** (2017), 369-383.

[Pu] P. PUCCI, *Critical Schrödinger–Hardy systems in the Heisenberg group*, to appear in Discrete Contin. Dyn. Syst. Ser. S, Special Issue on the occasion of the 60th birthday of Vicentiu D. Rădulescu.

Schrödinger-Poisson systems with critical growth

Riccardo Molle
Università degli studi di Roma "Tor Vergata"

This talk deals with elliptic problems of the form

$$\begin{cases} -\Delta u + V(x)u + K(x)\phi(x)u = u^5 \\ -\Delta\phi = K(x)u^2 \qquad x \in \mathbb{R}^3. \end{cases} \tag{SP}$$

Problem (SP) exhibits a "double" lack of compactness because of the unboundedness of $\mathbb{R}^3$ and the critical growth of the nonlinear term and in our assumptions ground state solutions of (SP) do not exist. We consider both $\lim_{|x|\to\infty} V(x) = 0$ and $\lim_{|x|\to\infty} V(x) > 0$ and we prove existence and multiplicity of bound state solutions. Let us remark that problem (SP) reduces to a Schrödinger equation when $K \equiv 0$: in this case we get a new result. This is a joint work with Giovanna Cerami.

Solutions of mountain pass type for double well potential systems

Piero Montecchiari
Università Politecnica delle Marche

We study a Hamiltonian system possessing a double well potential for which the existence of heteroclinic and homoclinic solutions that are local minimizers of an associated functional is known. Under an additional mild non-degeneracy condition on the set of all homoclinic and heteroclinic solutions, the existence of further heteroclinic and homoclinic solutions that are of mountain pass type is established through the use of a variant of the Mountain Pass Theorem.

Evolution problems in Perugia

Dimitri Mugnai
Università della Tuscia

We will present some recent results on evolution problems obtained by the PRIN team in Perugia. Different frameworks and different types of results will be presented, among them: existence, non existence, stability, Carleman estimates.

Embedded loops in the hyperbolic plane with prescribed, almost constant curvature

Roberta Musina
Università di Udine

Given a constant $k > 1$ and a real valued function K defined on the hyperbolic space $\mathbf{H}^2$, we study the problem of finding, for $\varepsilon \approx 0$, a closed and embedded curve u^ε in $\mathbf{H}^2$ having geodesic curvature $k + \varepsilon K(u^\varepsilon)$ at each point.

Index and stability of closed geodesics on semi-Riemannian manifolds

Alessandro Portaluri
Università di Torino

A celebrated result due to Poincaré asserts that a closed minimizing geodesic on a orientable surface is linearly unstable when considered as orbit of the co-geodesic flow. In this talk, starting from this classical theorem, we discuss some recently new results on the instability and hyperbolicity of closed (maybe not minimizing) geodesics of any causal character on higher dimensional (even not orientable) semi-Riemannian manifolds. Dropping the non-positivity assumption of the metric tensor is a quite challenging task since the Morse index is truly infinite.

This is a joint work with Xijun Hu and Ran Yang.

Small quasi periodic solutions for a class of quasi-linear dispersive PDEs on the circle

Michela Procesi
Università di Roma Tre

I shall discuss existence and linear stability of small quasi-periodic solutions for quasi linear dispersive PDEs on the circle; I shall particularly concentrate on the DP equation which is an integrable Hamiltonian system with asymptotically linear dispersion law. I will give an overview of the general strategies as well as of the difficulties in dealing specifically with the DP equation. This is based on joint work with R. Feola and F. Giuliani.

Mean curvature flow solitons

Marco Rigoli
Università di Milano

We define a very general notion of flow soliton and we study in some details the case where the generating vector field is closed conformal. In particular we consider warped product targets and translational solitons. Our qualitative results are, basically, derived with the help of appropriate maximum principles or via spectral considerations.

Intertwining solutions for magnetic relativistic Hartree type equations

Simone Secchi
Università degli Studi di Milano Bicocca

We prove the existence of infinitely many intertwining solutions to the magnetic pseudo-relativistic Schrödinger equation

$$\sqrt{(-i\nabla - A(x))^2 + m^2}u + V(x)u = (I_\alpha * |u|^p)\,|u|^{p-2}u, \quad \text{in } \mathbb{R}^N$$

under suitable assumptions. We assume in particular that A and V are symmetric with respect to a closed subgroup G of the group $O(N)$ of orthogonal linear transformations of $\mathbf{R}^N$.

The nodal set of solutions to some sublinear equations

Nicola Soave
Politecnico di Milano

We are concerned with the nodal set of solutions to sublinear equations of the form

$$-\Delta u = \lambda_+ \left(u^+\right)^{q-1} - \lambda_- \left(u^-\right)^{q-1} \quad \text{in } B_1$$

where $\lambda_+, \lambda_- > 0$, $q \in [1,2)$, $B_1 = B_1(0)$ is the unit ball in $\mathbb{R}^N$, $N \geq 2$, and $u^+ := \max\{u,0\}$, $u^- := \max\{-u,0\}$ are the positive and the negative part of u, respectively. This class includes, the two-phases *unstable obstacle problem ($q = 1$)*, as well as *sub-linear* equations. In both cases the right hand side is not locally Lipschitz continuous as function of u.

In this talk we present results regarding:

(a) the validity of the unique continuation principle; (b) the finiteness of the vanishing order at every point and the complete characterization of the order spectrum; (c) a weak non-degeneracy property; (d) the partial regularity of the nodal set of any solution: the nodal set is a locally finite collection of regular codimension one manifolds up to a residual singular set having Hausdorff dimension at most $N-2$ (locally finite when $N = 2$) and a partial stratification theorem.

Ultimately, the main features of the nodal set are strictly related with those of the solutions to linear (or superlinear) equations, with two remarkable differences. First of all, the admissible vanishing orders can not exceed the critical value $2/(2-q)$. At threshold, we find a multiplicity of homogeneous solutions, yielding the *non-validity* of any estimate of the $(N-1)$-dimensional measure of the nodal set of a solution in terms of the vanishing order.

The proofs are based on Weiss-type monotonicity formulæ, blow-up arguments and the classification of homogenous solutions.

The talk is based on joint works with Tobias Weth and Susanna Terracini.

On s-harmonic function on cones

Giorgio Tortone
Università di Torino

We deal with non negative functions satisfying

$$\begin{cases} (-\Delta)^s u_s = 0 & \text{in } C \\ u_s = 0 & \text{in } \mathbb{R}^n \setminus C, \end{cases}$$

where $s \in (0,1)$ and C is a given cone on $\mathbb{R}^n$ with vertex at zero. We consider the case when s approaches 1, wondering whether solutions of the problem do converge to harmonic functions in the same cone or not. Surprisingly, the answer will depend on the opening of the cone through an auxiliary eigenvalue problem on the upper half sphere. These conic functions are involved in the study of the nodal regions in the case of optimal partitions and other free boundary problems and play a crucial role in the extension of the Alt-Caffarelli-Friedman monotonicity formula to the case of fractional diffusions.

This is a joint work with Susanna Terracini and Stefano Vita.

Some characterizations of spherical sectors inside cones

Giulio Tralli
"Sapienza" Università di Roma

We consider partially overdetermined problems in conical domains and constant mean curvature hypersurfaces with boundary suitably attached to a smooth cone. For the case of convex cones, we present respectively a Serrin-type and an Aleksandrov-type result. We will focus on the role in the proofs of the convexity of the cone, and we will show a rigidity result in starshaped sectors related to non-convex cones. This is a joint work with F. Pacella.

Local regularity for degenerate and singular elliptic equations

Stefano Vita
Università di Torino

We deal with elliptic operators

$$-\operatorname{div}\left(|y|^a \nabla u\right),$$

which can be degenerate if $a > 0$ and singular if $a < 0$ on the "characteristic manifold" $\{y = 0\}$, while the Laplacian case is recovered for $a = 0$. We set the problem in $\mathbb{R}^n$ with variable $z = (x,y)$ such that $x \in \mathbb{R}^{n-1}$ and $y \in \mathbb{R}$. We are interested in the local regularity for solutions of some problems in which this operator is involved.
